MONTHLY NOTICES

OF THE

ROYAL ASTRONOMICAL SOCIETY

Vol. 93 No. 3

MEETING OF 1933 JANUARY 13

Dr. H. Knox-Shaw, President, in the Chair.

The election by the Council of the following Fellows was duly confirmed :—

George Barron, B.Sc., The Grammar School, Morley, Leeds ;
Frank M. Bateson, 368 Main Road, Karori, Wellington, New Zealand ;
Ernest William Bliss, Demonstrator in Meteorology, Imperial College of Science and Technology, South Kensington, S.W. 7 ;
Keith Edward Bullen, M.A., B.Sc., St. John's College, Cambridge ;
John Edmund Calthrop, M.A., M.Sc., Lecturer and Demonstrator in Physics, East London College, Mile End Road, London, E. 1 ;
Don Tudor Ernest Dassanayake, B.Sc., Colombo Observatory, Ceylon ;
J. A. Edgar, B.A., Sidney Sussex College, Cambridge ;
Major Edward Aubrey Glennie, D.S.O., R.E., Geodetic Branch, Survey of India, Dehra Dun, India ;
Garret A. Hobart, 129 Market Street, Paterson, N.J., U.S.A. ;
George Lyman, B.Sc., 21 Holt Road, Blackheath, Birmingham ;
Arthur Gordon Nock, B.Sc., Beechcroft, Bloxcidge Street, Langley Green, Birmingham ;
Clifford E. Smith, State Teachers College, Fresno, California, U.S.A. ;
Bertha Swirles, Ph.D., Department of Mathematics, Imperial College of Science and Technology, South Kensington, London, S.W. 7 ; and
Geoffrey Gilbert Wiles, M.Sc., Balliol College, Oxford.

The following candidates were proposed for election as Fellows of the Society, the names of the proposers from personal knowledge being appended :—

C. J. M. Aanensen, 165 Moorland Road, Splott, Cardiff (proposed by J. Jackson) ;

S. Chandrasekhar, Trinity College, Cambridge (proposed by E. A. Milne) ;

Gabriel Gideon Cillié, Hertford College, Oxford (proposed by E. A. Milne) ;

Raymond Edward Victor Ely, Salcombe Lodge, Ewell, Surrey (proposed by F. W. Dyson) ;

James de Graaff Hunter, M.A., Sc.D. (late Director of the Geodetic Branch, Survey of India), 9 Westminster Park, Chester (proposed by G. P. Lenox-Conyngham) ;

Rev. Franklin Isaac Hutchinson, D.Litt., B.Ph., The Rectory, Barningham with Coney Weston, near Bury St. Edmunds (proposed by F. W. Dyson) ;

Harald von Klüber, Dr.Phil., Einstein-Institut, Potsdam, Germany (proposed by E. A. Milne) ;

Percy George Murden, 267 High Holborn, London, W.C. 1 (proposed by A. R. Hinks) ; and

Harry B. Rumrill, Conestoga Road, Berwyn, Pennsylvania, U.S.A. (proposed by J. Stokley).

Ninety presents were announced as having been received since the last meeting, including :—

British Association Mathematical Tables, vol. ii, Emden Functions (presented by the British Association Tables Committee).

The President announced that the Gold Medal of the Society had been awarded by the Council to Dr. V. M. Slipher, Director of the Lowell Observatory, for his spectroscopic researches on planets, stars and nebulæ.

CHANGE FROM 6″ to 5″ IN THE REDUCTION OF THE OCCULTATIONS OBSERVED IN 1933.

Ernest W. Brown and Dirk Brouwer.

Beginning with the first lunation of 1933 (New Moon, 1932 December 27) it is requested that 5″ be added to the Moon's mean longitude instead of 6″ as used during the past four years. This is equivalent to the addition of $0{\cdot}152\Delta\alpha$, $0{\cdot}152\Delta\delta$ where $\Delta\alpha$, $\Delta\delta$ are the variations per minute, or to that of $0^{h}{\cdot}00253$ to the time at which the Moon's position is obtained from its ephemeris. In communicating results, the amount added should be stated.

A NEW METHOD OF USING A SPECTROGRAPH FOR SOLAR ROTATION WORK.

J. Evershed, F.R.S.

In his valuable paper entitled "Spectroscopic Observations of the Sun's Rotation made at the Royal Observatory, Edinburgh, 1914–31," * Mr. Storey calls attention to the "large differences in the rotation value which occur from plate to plate and often on the same plate." As I have had similar experiences in measuring rotation spectra, it seems desirable to attack the problem once again with improved arrangements for obtaining the photographs and a more accurate method of measuring them. Also the present time of minimum solar activity affords a favourable opportunity for obtaining consistent values of the rotation uninfluenced by sunspot or other disturbances. It is evidently important to discover whether the apparent variations are real or due to instrumental causes.

I give here a description of the apparatus I am now using, and the methods of measuring the spectra.

The spectrograph at Ewhurst is fed by a cœlostat which reflects the Sun southward. A second mirror sends the light downwards through a 6-inch object-glass of 20 feet 9 inches focal length. A third mirror in the underground room reflects the light horizontally northward to a focus on the spectrograph slit. By this arrangement the east and west points on the Sun's image lie on a horizontal diameter, and remain so at all times.

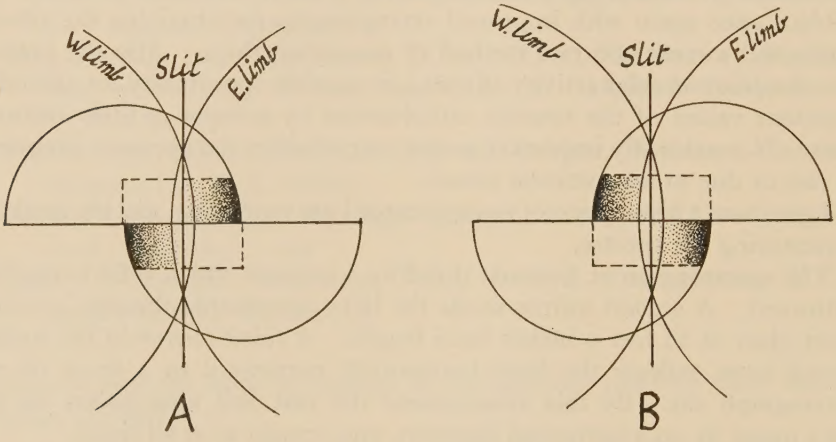
The beam of sunlight coming from the object-glass is divided into two parts about 3 feet from the focus, where two right-angle prisms are placed, reflecting the light from the east and west limbs upwards to a single plane mirror: this reflects it to the slit of the spectrograph. The two images, east limb and west limb, are here in focus and may be moved relatively until they coincide or overlap slightly. This is done by means of a fine screw adjustment applied to each of the two reflecting prisms moving one image horizontally through a solar diameter and adjusting the other image vertically.

The object is to photograph the two limbs simultaneously; but it is necessary to cut out one-half of each to avoid overlapping of the two spectra. Also on account of the considerable angle between the two beams of light (the angle subtended at the slit by the two patches of light falling on the reflecting prisms) neither beam will illuminate the collimator lens, and they fall on each side of the collimator tube. To correct this and at the same time to cut out one-half of each limb, a thin plano-convex cylindrical lens (a spectacle lens is used) is cut in half, forming essentially two similar wedges of varying angle. Each half is placed over half the length of the slit, and they are carefully adjusted laterally, as shown in the diagram A. By this arrangement the light of half of the east limb image and half of the west

* *M.N.*, **92**, 737, 1932.

image is made to fall centrally on the collimator lens. The other halves of the limb images are by the same action thrown entirely away from the collimator, and are completely suppressed by the velvet lining of the tube. In this way the spectra of the two limbs appear in contact, and the displacements of the spectrum lines at the junction of the images represent twice the Sun's rotation velocity at the latitude of the east and west points, which may vary between the equator and latitude 26°.

In the diagram B a similar device is shown, but with the half lenses arranged to interchange the east and west limbs and so give the displacements between the upper and lower spectra in the opposite direction. The purpose of this is to facilitate the measurement of the spectra, as will appear later. In the diagrams the portion of each half lens that is effective as a



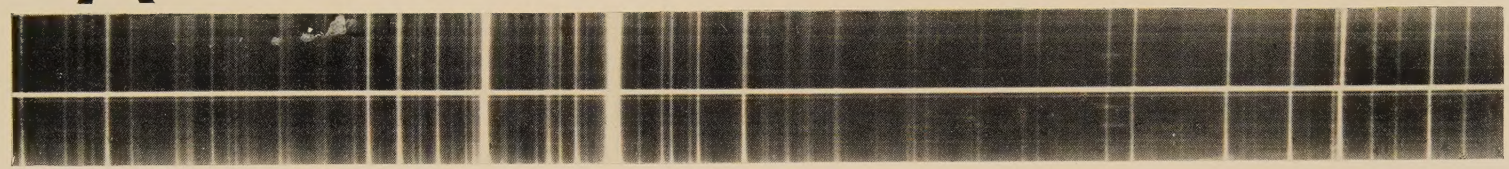
wedge is shown by the broken lines, the shaded part representing the thinner part of the cylindrical surface.

This method of obtaining east and west spectra simultaneously supersedes that in which small reflecting prisms are arranged at the two limbs, reflecting the light to similar prisms placed over the slit at the centre of the Sun's image, a method which is difficult to arrange satisfactorily except for very large solar images. The equal illumination of the collimator, or prism face, by the two limbs is of course essential, and is much more easily secured with the variable wedges than with the small reflecting prisms. The half lenses are cemented to a metal plate which can be easily attached to or detached from the slit mounting. The cement used * takes a day or two to harden if no heat is applied, and this property makes it easy to slide each half to the correct position, determined beforehand approximately, so that the light from each limb falls centrally on the collimator. With the slit at its working width 0·02 mm. the first diffraction band is much wider than the effective aperture of the prisms; consequently any slight error in the adjustment of the wedges has no effect on the spectra.

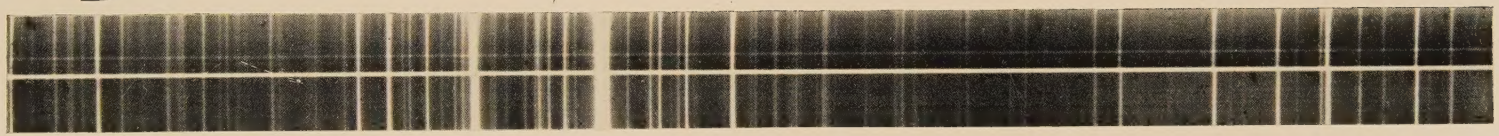
In solar rotation work it is best to avoid taking the light from points very

* This has the trade name " Firmas."

A



B



Solar Rotation Plate. Latitude 10°·8. D region (red to left).

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near to the limb, owing to the relative increase in the admixture of scattered light from the sky as the limb is approached: it is necessary, therefore, to know the exact distance within the limb from which the light is taken in order to determine the correction to be applied. This is not easily ascertained when small prisms are used, but in the new method the distance between the cusps in the overlapping images affords an accurate measure of the position of the slit within the limbs at the east and west points.

In order to measure the spectra with the greatest possible accuracy, and to avoid systematic errors, the positive on negative method is adopted.* By this scheme the intervals measured are doubled, and so are equivalent to four times the rotational velocity. Also, with practice, greater precision can be achieved in setting the positive image of a line on the negative image than is possible in placing a fine thread centrally on a line. This applies especially to broad lines such as those of sodium and magnesium.

In measuring two contiguous spectra by the positive on negative method it is necessary to obtain *reversed* positives of the spectra, and as satisfactory copies cannot easily be obtained by direct contact through the glass a simple method is to photograph a pair of negatives, one with the cylindrical lenses arranged as in diagram A, and the other with another pair of half lenses arranged as in B. Ordinary contact copies are then taken of the two negatives, and in the measuring micrometer the positive of A is superposed on the negative of B without reversing the spectra end for end. This enables displacements to be measured to the best advantage by this method, and eliminates a troublesome systematic error which may arise if the spectrum lines are not exactly normal to the spectrum, a condition that requires very careful adjustment of the slit mounting. This error applies equally to the ordinary method of measuring when the thread is not quite parallel to the lines.

In order to reduce the accidental errors of measurement, the positive image B may be superposed on the negative A, and the mean of the two sets of measures taken.

In cases where, owing to cloudy conditions, only one east and west spectrum has been obtained, this is measured by taking a reversed positive in a special 8-foot camera in which the negative is placed film outwards, and an image formed in the conjugate focus of a lens placed exactly midway between film and plate. In this case it is necessary to ascertain that the lines are at right angles to the spectrum. This is tested by placing a reseau ruled in squares over the negative.

A considerable number of spectra near D and near *b* has been secured by this method, using the high-dispersion liquid prism spectrograph, and the measures of these give very promising results.

A sample photograph of the D region is given in Plate 7 showing the opposite displacements in the images A and B. Incidentally these spectra, representing latitude 10°·8, show how small is the interval to be measured—about the width of the narrower lines—and we attempt to measure this to an accuracy of about one part in two hundred.

* *Kodaikanal Observatory Bulletin*, 3, 17, 1913.

It is hoped to obtain a good series of these spectra during the period of minimum solar activity, and to communicate the results of the measures to the Society in a subsequent paper.

Ewhurst :
1932 *December* 27.

OBSERVATIONS OF SOLAR FLOCCULI MADE WITH THE SPECTROHELIOSCOPE AT THE ROYAL OBSERVATORY, GREENWICH, DURING THE YEAR 1932.

(Communicated by the Astronomer Royal)

This note summarises the observations, made with the spectrohelioscope during the year 1932, of bright and dark flocculi seen on the Sun's disc in *Hα* light. Observations were commenced regularly with this instrument in 1930, and summaries for the years 1930 and 1931 are given in *M.N.*, **91**, 593, 1931, and **92**, 438, 1932 respectively.

The spectrohelioscope, which has been lent to Greenwich by the Mount Wilson Observatory, is used in conjunction with a Rowland grating ; the first-order spectrum was used throughout the year, the scale being about 1 mm. = 4 A.

Measures of radial velocity are made with the "line-shifter," the scale of which is from 0–10 divisions = 0·37 A. = 17 km./sec. at *Hα*. The zero of the *Hα* line is determined from measures of the darkest part of the line from an undisturbed portion of the Sun near the centre of the disc. All measured displacements are interpreted as being due to the effect of radial velocity. The measures have not been corrected for the effect of the Sun's rotation or of the Earth's orbital and rotational velocity.

The probable error of a single measure of radial velocity, as determined from a number of successive measures, is about 3 km./sec., including the probable error of the zero determination. Four measures being usually made on each flocculus, the probable errors of the tabulated values are of the order of 2 km./sec.

Observations were made on 171 days during the year 1932 ; on about 110 days the survey of the Sun's disc was complete or nearly so. In the course of searching for active areas on the Sun, measures of radial velocity were made systematically on all flocculi, both bright and dark. Summaries of these measures are given in the following tables, which may be compared with those for the years 1930 and 1931 in the summaries already referred to. The observations have been classified as follows :—

(*a*) Areas of bright *Hα* flocculi.
(*b*) Dark *Hα* flocculi not associated with sunspots or bright areas.
(*c*) Dark *Hα* flocculi associated with sunspots.

(*d*) Dark *Hα* flocculi associated with bright areas not containing sunspots on the day of observation.

(*e*) Unusually bright *Hα* flocculi.

It will be seen from Tables I to IV that the flocculi as classified above are differentiated by their radial velocities as well as by their characteristics in appearance and occurrence.

(*a*) Measures were made of 106 areas of bright flocculi in the central part of the Sun's disc, contained by a circle of diameter half that of the disc. The average measured displacement of the wave-length at the position of their maximum intensity from the centre of the *Hα* line is equivalent to a radial velocity of −0·6 km./sec.

Table I gives the frequency of the measured values ranging from −6 to +4 km./sec., a minus sign indicating an outward motion from the Sun and a plus sign an inward motion. For purposes of comparison, measures of 157 bright flocculi outside the central part of the disc are also tabulated, the mean average displacement of these being also equivalent to −0·6 km./sec.

TABLE I

Measures of Radial Velocity of Bright Hα Flocculi, 1932 *and* 1930–32

Velocity km./sec.	Number, 1932		Number, 1930–32	
	Central	Non-central	Central	Non-central
−11	...	...	2	0
10	...	...	3	2
9	...	...	2	2
8	...	1	4	1
7	...	1	4	13
6	1	1	10	15
5	4	10	21	25
4	2	11	20	26
3	10	23	35	44
2	19	17	45	49
−1	18	21	52	41
0	18	21	52	66
+1	21	19	46	43
2	7	12	22	27
3	5	10	14	21
4	1	5	4	10
5	...	2	2	5
6	...	0	1	1
7	...	0	...	2
8	...	1	...	1
9	...	0	...	0
10	...	1	...	2
11	...	...	...	1
12				
13				
14				
+15	...	1	...	1
Means (km./sec.)	−0·6	−0·6	−1·4	−1·1

The fourth and fifth columns of Table I give the frequencies of the measured values for the two parts of the disc for the three years 1930–32 as a whole.

Table I shows that the bright *Hα* flocculi are not associated with large vertical movements in the sense usually applicable to chromospheric phenomena. The distribution of the velocities corresponds essentially to that of a probable error of 2 km./sec. Whether any significance can be attached to the small mean outward motion for the bright flocculi cannot be said without measures of greater precision.

With respect to the reality of the largest values, it is perhaps significant that there is some tendency for them to be measured in the outer part of the disc, thus suggesting that lateral velocities are occasionally measured near the Sun's limbs.

The mean radial velocity of bright flocculi for each of the three years 1930–32 are as follows :—

Number	Central	Non-central
1930	−2·2 km./sec.	−2·2 km./sec.
1931	−1·1 ,,	−0·7 ,,
1932	−0·6 ,,	−0·6 ,,

The solar activity has fallen considerably during the above period. Whether the corresponding decrease in the mean measured radial velocity of the bright *Hα* flocculi is related to this or is merely spurious, can only be decided by continuing the observations throughout a sunspot cycle.

(*b*) Measures were made of 49 dark flocculi not associated with sunspots or bright *Hα* flocculi. These were usually conspicuous dark markings a few minutes of arc in length and tending to avoid sunspots. It was characteristic of them that, as the line-shifter was turned, they appeared or disappeared as a whole, thus exhibiting no appreciable progression of radial velocity along their length. In duration they approximated to that of sunspots. Since beginning observations with the spectrohelioscope in 1930 they have become less numerous, and the very extensive, dense ones rather rare.

The mean of the measured radial velocities of these 49 flocculi was −0·6 km./sec. Table II gives the frequency of the measures for these and for the flocculi in the non-central part of the disc, the latter numbering 243 and giving a mean measured radial velocity of −0·1 km./sec. The observations for the years 1930–32 combined are also given in the last two columns.

It may be concluded from Table II that the dark *Hα* flocculi not associated with sunspots or bright areas have, in general, scarcely measurable radial movements. The few large velocities tabulated are, however, certainly real, and appear to represent statistically a rather infrequent and brief eruptive phase of some of these dark flocculi. Their behaviour is in marked contrast to that of flocculi associated with sunspots and summarised in Table III.

(*c*) Table III gives a tabulation of measured radial velocities of 166 dark flocculi that were observed in close proximity to sunspots, to which many

TABLE II

Measures of Radial Velocity of Dark Hα Flocculi not associated with Sunspots or Bright Flocculi, 1932 and 1930–32

Velocity km./sec.	Number, 1932		Number, 1930–32	
	Central	Non-central	Central	Non-central
−31–35	...	...	...	1
26–30	...	...	...	3
21–25	...	...	1	1
16–20	...	1	...	3
11–15	1	...	6	7
10	...	...	1	3
9	...	...	2	7
8	...	...	5	9
7	...	1	8	14
6	...	4	15	20
5	1	3	12	29
4	1	6	16	22
3	1	21	21	56
2	10	20	37	74
− 1	11	47	34	103
0	14	44	55	130
+ 1	2	39	32	91
2	4	25	24	79
3	3	18	21	56
4	...	11	11	35
5	1	2	2	20
6	...	...	2	8
7	...	...	2	5
8	...	...	...	5
9	...	...	...	2
10	...	...	...	2
11–15	...	...	...	1
16–20	...	...	1	3
21–25	...	...	...	1
+26–30	...	1	...	3

of them, judging from the character of their motion, were evidently related. The flocculi of this class were generally short-lived (those with the larger velocities lasting for half an hour or less) and were not so extensive in area as those of class (*b*), the average length being about $\frac{3}{4}'$. Several were small, dense, circular markings about 20″ in diameter. Progression of radial velocity along the length of the filamentary markings were observed in about 25 per cent. of the cases. For the purpose of this tabulation the largest measured value of such observed progression of velocity has been taken in each case irrespective of sign. The detailed measures will be published in the *Greenwich Photoheliographic Results* for 1932, in continuation of those already published in the volumes for 1930 and 1931.

An inspection of Table III shows the active nature of a large proportion of the dark flocculi observed in association with sunspots. A maximum occurs between velocities of 20–50 km./sec. The signs of the velocities in Table III show an excess of inward motions.

TABLE III

Measures of Radial Velocity of Dark Hα Flocculi associated with Sunspots, 1932

Velocity km./sec.	Number	Velocity km./sec.	Number
0–4	43	50–59	5
5–9	3	60–69	3
10–19	12	70–79	2
20–29	36	80–89	3
30–39	39	90–99	1
40–49	18	110	1

(*d*) The activity of dark flocculi referred to in Table III seems to depend to a large degree upon the presence of sunspots, however small, and not usually upon the presence of bright flocculi alone, which are invariable accompaniments of sunspots. This is suggested by Table IV, which gives the frequencies of radial velocities of dark *Hα* flocculi observed during 1932 associated with areas of bright *Hα* flocculi, but in which no sunspot was recorded at Greenwich at or about the time of observation of the flocculus.

TABLE IV

Measures of Radial Velocity of Dark Hα Flocculi associated with Bright Flocculi but not with Sunspots, 1932

Velocity (km./sec.)	0–4	5–9	10–19	20–29	30–39	40–49
Number	33	3	2	4	4	1

Although in Table IV there are a few velocities between 20 and 40 km./sec., the numbers are small compared with those in Table III.

(*e*) Ten small eruptions of bright *Hα* flocculi were observed during 1932, and are tabulated below in continuation of similar tables given for 1930 and 1931 in *M.N.*, loc. cit. The unit of area is one millionth of the Sun's hemisphere. The last column contains the date of any magnetic disturbance recorded at Abinger (in which the ranges in $D \geq 30'$ or in H or $V \geq 150\gamma$) within 4·0 days of the time of central meridian passage of the disturbed area (given in the second column of Table V).

These very bright flocculi, appearing suddenly and lasting for quarter or half an hour, have no appreciable radial velocity, but the dark transient flocculi usually associated with them have rapid motion, the largest radial velocity (110 km./sec.) being measured on October 6.

During 1932 an attempt was made to measure the relative brightness of *Hα* flocculi, especially those comprising the bright eruptions. Measures of light intensity were therefore made on any day possible when bright flocculi were visible on the disc. For the purpose, a neutral wedge was inserted in the field of view between the rotating prisms of the spectrohelioscope and the eyepiece, and a comparison was made between the light intensity of the flocculus and that of the general disc immediately surrounding it, by sliding the wedge until the flocculus (seen through the wedge) appeared of equal brightness to that of the adjacent background (seen through

TABLE V

Eruptions of Bright Hα Flocculi, 1932

Date of Eruption	Associated Sunspot			Magnetic Storm Commenced
	C.M.P.	Lat.	Area	
Jan. 26·5	Jan. 23·5	6° S.	24	None
Mar. 2·4, 2·5	Feb. 27·0	5° S.	293	Feb. 23·7
Mar. 7·5, 7·5	Mar. 4·4	12° N.	244	None
Apr. 22·4	Apr. 25·4	10° N.	545	None
May 25·4	May 26·4	4° N.	314	May 29·5
July 30·4	Aug. 2·5	8° S.	215	None
Aug. 28·4	Aug. 25·4	6° S.	50	Aug. 27·6
Oct. 6·4	Oct. 6·9	11° N.	2	None
Oct. 18·5	Oct. 24·4	10° N.	140	Oct. 20·5
Dec. 3·5	Dec. 5·0	5° N.	100	None

the clear glass plate on which the wedge was mounted). The wedge was calibrated on the microphotometer used in connection with colour-temperature work at the Observatory. The frequencies of the measured intensities of 125 flocculi, including 9 bright eruptions, are given in Table VI, the light intensity of the adjacent background being unity in each case. It will be seen from the table that the most frequent light ratio for flocculus to background is 1·4.

TABLE VI

Measured Light Intensities of Bright Hα Flocculi, 1932

Light Intensity	Ordinary Flocculi		Eruptions	
	Central	Non-central	Central	Non-central
1·1	7	7		
1·2	4	9		
1·3	9	12		
1·4	16	18		
1·5	6	10		
1·6	...	5		
1·7	1	3		
1·8	...	2		
1·9	1	3	...	1
2·0	...	2 *	...	3
2·1	...	1 †	1	
2·2	...	...	2	
2·3				
2·4	...	...	...	1
...				
3·2	...	...	...	1 †

* One of these flocculi was very near the Sun's limb; the other may have represented a bright eruption, but observations were incomplete on May 21.

† Flocculi near Sun's limb.

Royal Observatory, Greenwich:
1933 *January* 9.

LARGE SOLAR DISTURBANCE OF 1932 DECEMBER 12.

C. P. Butler.

(Communicated by the Director, Solar Physics Observatory, Cambridge University)

After an interval of slight activity a disturbance of considerable magnitude developed during the second week of December, being probably a return of a previous formation which appeared during the third week of November. On December 6, as it came round the east limb, it presented no marked signs of bright eruptions, the chief feature being three prominent dark markings radiating from the spot centre on the northern side.

No further observations were possible at Cambridge until December 12, when a spectroheliogram was obtained in *Hα* light through thin cloud at $11^h\ 40^m$ G.C.T. The spot umbra (long. 325°; N. lat. 10°) was then multiple, with large elliptical penumbra bounded by a well-marked bright ring. There was no bright flocculus immediately surrounding the penumbra, but some distance away a slender semicircular arc of bright material was almost concentric with the spot centre. This type of bright formation is that frequently associated with special activity during periods of eruption.

Outside this there was a long thin dark filament, at first running N.E. for about 10°, and then curved northward. On the S.W. side of the spot there is indication of a prolongation of the dark filament, almost in line with the straight portion on the N.E. side. Both filaments terminate abruptly at the edge of the prominent bright penumbral ring.

Contrasting with the general mottling of other portions of the solar disc, there is a well-marked active area, partly of vortical structure, extending over an area about 30° in diameter, which would indicate a disturbance of outstanding magnitude for the present phase of the solar cycle.

No further records were possible up to the time of passing over the west limb, but it is of interest to note that about 52 hours afterwards a large magnetic disturbance was recorded at the Greenwich Magnetic Station, about 1932 December $14^d\ 16^h$, and it will be helpful if records from other stations permit examination of the history of this disturbed area.

Further photographs obtained on 1933 January 9 show two small spots in similar longitude to the large disturbance of December, but in slightly higher latitude, which may be the result of breaking up of the large group. Small areas of bright eruptive material are shown in the region between the two-spot umbræ. Attention may be called to the forked end of the N.E. dark marking, with the point of the arrow at the margin of the bright penumbral ring.*

Most previous notes of bright rings in connection with sunspots appear

* Hale, *Ap. J.*, **23**, 62, 1906.

N

E W

S

Disturbance round Sun-spot, 1932 Dec. $12^{d}\ 11^{h}\ 40^{m}$.

Butler, Large Solar Disturbance of 1932 December 12.

to refer to a feature immediately surrounding the umbra, and Chevalier* specifically mentions this position; the bright ring above noted is round the *outer* edge of the penumbra. Some reserve may be necessary as to the possibility of the appearance being partly produced photographically at junctions of bright and dark areas of developed silver bromide.†

THE COMPUTATION OF TOTAL SOLAR ECLIPSES.

L. J. Comrie.

This paper attempts to adapt, for the purposes of a computer using calculating machines, Chauvenet's classical formulæ for computing the central line of a central eclipse, and also provides formulæ for the rigorous computation of the northern and southern limits of totality or annularity. The more one reads Chauvenet's chapter on eclipses, the more one is impressed with his masterly thoroughness, and, in fact, his methods have been used unchanged for seventy years. However, a modern computer, if working with a calculating machine, does not require the auxiliary angles that characterise adaptation to logarithms, but desires to do his work with pure numbers, and with the minimum possible references to trigonometrical tables.

The notation used is that of Chauvenet, except that i is replaced by $\tan f$, μ_1 by μ, and ω (longitude) by λ. The definitions of the published Besselian elements are given each year in the *Nautical Almanac*, and need not be repeated here. In addition the following unpublished elements, in which accents denote hourly variations, are required:—

$$a' = -l' - x \tan f\, \mu' \cos d \quad (\mu' \text{ in radians})$$
$$b' = -y' + x\mu' \sin d$$
$$c' = x' + y\mu' \sin d + l \tan f\, \mu' \cos d$$
$$d' = \text{hourly variation of } d, \text{ in radians}$$
$$y_1 = \frac{y}{\rho_1} \qquad \text{Also } \frac{l}{\rho_1}$$
$$\sin d_1 = \frac{\sin d}{\rho_1} \qquad \cos d_1 = \frac{0{\cdot}996633 \cos d}{\rho_1}$$

in which $\rho_1^2 = 1 - e^2 \cos^2 d$, where e is the eccentricity of the Earth's meridian. If c, the compression, is taken as $\frac{1}{297{\cdot}0}$, then, since $e^2 = 2c - c^2$, the following table results :—

* Chevalier, *Ap. J.*, 24, 278, 1906.
† Royds, *M.N.*, 85, 465, 1925.

[Table

d	$\frac{1}{\rho_1}$		d	$\frac{1}{\rho_1}$		d	$\frac{1}{\rho_1}$	
°			°			°		
± 0	1·003378	1	± 8	1·003313	18	± 16	1·003120	32
1	3377	3	9	3295	19	17	3088	34
2	3374	5	10	3276	21	18	3054	36
3	3369	7	11	3255	23	19	3018	37
4	3362	10	12	3232	25	20	2981	39
5	3352	11	13	3207	27	21	2942	40
6	3341	13	14	3180	29	22	2902	42
7	3328	15	15	3151	31	23	1·002860	
8	1·003313		16	1·003120				

The elements d_1, d_2, ρ_2, e, E, f, F, used by Chauvenet, are not required. The above elements, together with those published, are computed for the principal hours of the eclipse, checked by differencing, and interpolated as required; they should be taken to 5 decimals. In the case of x and y there will be unavoidable roughnesses of several units in the last decimal, and it is advisable, to facilitate subsequent checking, to smooth these values. Suppose a series of quantities to have been computed for 5 or 7 integral hours, and that it is required to represent them by a parabola

$$y = a + bx + cx^2$$

in which equation the letters used are not to be confused with those in the eclipse geometry, and in which x is measured from the middle point. Then the result of fitting by least squares is

	5-point curve	7-point curve
a	$\frac{17[y] - 5[x^2y]}{35}$	$\frac{7[y] - [x^2y]}{21}$
b	$\frac{[xy]}{6}$	$\frac{[xy]}{28}$
c	$\frac{[x^2y] - 2[y]}{14}$	$\frac{[x^2y] - 4[y]}{84}$

Chauvenet's formulæ for the central line are

$$\sin\beta \sin\gamma = x \qquad c \sin C = y_1$$
$$\sin\beta \cos\gamma = y_1 \qquad c \cos C = \cos\beta$$

$$\cos\phi_1 \sin\theta = x$$
$$\cos\phi_1 \cos\theta = c \cos(C + d_1)$$
$$\sin\phi_1 = c \sin(C + d_1)$$

$$\tan\phi = \frac{\tan\phi_1}{\sqrt{1 - e^2}} = 1{\cdot}00338 \tan\phi$$
$$\lambda = \mu - \theta$$

in which, besides the essential openings of trigonometrical tables at θ and ϕ,

there are four more openings at γ, β, C, $C+d_1$. In the following form these four openings are replaced by one in Barlow's *Tables*.

$$\cos\beta = \sqrt{1 - x^2 - y_1^2}$$

$$\cos\phi_1 \sin\theta = x$$
$$\cos\phi_1 \cos\theta = -y_1 \sin d_1 + \cos\beta \cos d_1$$
$$\sin\phi_1 = y_1 \cos d_1 + \cos\beta \sin d_1$$

Check : Sum of squares = 1

$$\tan\phi = 1{\cdot}00338 \tan\phi_1$$
$$\lambda = \mu - \theta$$

For the duration :

$$L = l - \tan f \cos\beta$$
$$a = c' - \mu' \cos d \cos\beta$$

$$\text{Duration} = \frac{7200\, L}{\sqrt{a^2 + b'^2}} \text{ seconds}$$

in which the suffix 2, indicating that the quantities relating to the umbra are under consideration, has been dropped. The approximate altitude at mid-totality may be found from

$$\sin(\text{altitude}) = \cos\beta$$

With regard to the northern and southern limits of total or annular eclipse, Chauvenet says: " These limits may be rigorously determined by the method of Art. 311 . . . but I here propose to deduce them from the previously computed curve of central eclipse. . . . This degree of approximation will in the greater number of cases suffice to determine points on the limits within 2′ or 3′, which is practically quite accurate enough." Chauvenet's approximate differential formulæ are still used by the ephemerides, which publish the limits to 0′·1, the last figure being utterly meaningless. In these more exacting days, particularly with eclipses that traverse thickly-populated countries, we can hardly be content with an accuracy of 3′, especially as attempts are made to determine the corrections to the latitude of the Sun and Moon by an exact determination, from observation, of the limits.

The difficulty in Chauvenet's rigorous method is that the values of one independent variable Q, and its dependent variable ζ, must be varied until an equation of condition is satisfied. As Chauvenet remarks, " this can be effected only indirectly, or by successive approximations." In the scheme now put forward the working is direct, although partially based on differentiations, and the results rigorous. Proofs are left to the reader. Using the central line calculation, with the upper sign applying to the northern limit, we have

$$\cos\beta_0 = \sqrt{1 - x^2 - y_1^2}$$

$$\sin Q_0 = \frac{b'}{\sqrt{a^2 + b'^2}} \quad \text{same sign as } b'$$

$$\cos Q_0 = \frac{a}{\sqrt{a^2+b'^2}} \quad \text{always positive}$$

$$p = b' - d' \cos \beta_0$$
$$q = a \mp \frac{\mu' \cos d \,.\, L}{\cos \beta_0}(x \sin Q_0 + y_1 \cos Q_0)$$

$$\Delta Q = \mp \frac{a' \mp p \cos Q_0 \pm q \sin Q_0}{\sqrt{a^2+b'^2}}$$

$$\sin Q = \pm (\sin Q_0 + \cos Q_0 \,.\, \Delta Q)$$
$$\cos Q = \pm (\cos Q_0 - \sin Q_0 \,.\, \Delta Q)$$

$$x - l \sin Q = A$$
$$y_1 - \frac{l}{\rho_1} \cos Q = B$$

$$\cos \beta = \sqrt{1 - A^2 - B^2}$$
$$\zeta = \cos \beta - \tan f (A \sin Q + B \cos Q)$$

As a check at this stage :—

$$p = b' - d' \cos \beta \quad \text{(as before, since } d' \text{ is always small)}$$
$$q = c' - \mu' \cos d \,.\, \zeta_1$$

Then $a' - p \cos Q + q \sin Q$ should not exceed $\pm$ 0·00010.

$$\xi = A + \zeta_1 \tan f \sin Q$$
$$\eta_1 = B + \zeta_1 \tan f \cos Q$$

$$\cos \phi_1 \sin \theta = \xi$$
$$\cos \phi_1 \cos \theta = -\eta_1 \sin d_1 + \zeta_1 \cos d_1$$
$$\sin \phi_1 = \eta_1 \cos d_1 + \zeta_1 \sin d_1$$

Check : Sum of squares = 1

$$\tan \phi = 1{\cdot}00338 \tan \phi_1$$
$$\lambda = \mu - \theta$$

In systematic calculations, the computer is greatly assisted by good draughtsmanship in the design of his computing form. A form suitable for the practical working of the above formulæ is shown below. Three parallel columns cover the northern limit, central line and southern limit respectively, their juxtaposition enabling the leading figures of many of the quantities to be checked by differencing by eye. In the column of precepts, three precepts indicate that separate quantities are given in the three columns ; where two precepts are given the first relates to the outer columns, and the second to the middle column. In the actual calculations, quantities given as whole numbers are in units of the 5th decimal. The order in which the various entries are to be made is easily determined from the formulæ. A choice of tan θ or cot θ is given, so that whichever is less than 1 may be chosen. The illustration is for the eclipse of 1940 October 1.

	Northern Limit	Central Line	Southern Limit
G.M.T.		$14^h\ 00^m$	
$\sin d_1,\ l,\ \tan f$	−0·05714	−0·01207	+0·00465
$\cos d_1,\ \frac{l}{\rho_1},\ \mu' \cos d$	+0·99837	−0·01211	+0·26145
$-l \sin Q,\ x$	+ 466	+0·63087	− 459
$x - l \sin Q = A$	+0·63553		+0·62628
$\zeta_1 \tan f \sin Q$	+ 111		− 108
Sum $= \xi = \cos \phi_1 \sin \theta$	+0·63664		+0·62520
$-\frac{l}{\rho_1} \cos Q,\ y_1$	+ 1117	−0·47120	− 1120
$y_1 - \frac{l}{\rho_1} \cos Q = B$	−0·46003		−0·48240
$\zeta_1 \tan f \cos Q$	+ 266		− 263
Sum $= \eta_1$	−0·45737		−0·48503
$\cos \beta,\ \cos \beta_0$	+0·62006	+0·61642	+0·61242
$-\tan f(A \sin Q + B \cos Q)$	+ 83		− 97
Sum $= \zeta_1$	+0·62089		+0·61145
$-\tan f \cos \beta_0$		− 286	
$l - \tan f \cos \beta_0 = L$		−0·01493	
b'		+0·16667	
$-d' \cos \beta_0$		+ 17	
Sum $= p$		+0·16684	
c'		+0·56337	
$-\mu' \cos d\ .\ \zeta_1,\ \ -\mu' \cos d \cos \beta_0$	−0·16233	−0·16117	−0·15986
Sum $= q,\ \ a$	+0·40104	+0·40220	+0·40351
$\mu' \cos d\ .\ L(x \sin Q_0 + y_1 \cos Q_0)$		+ 123	
$q_0 \cos \beta_0$	+0·40097		+0·40343
$\sqrt{a^2 + b'^2}$		+0·43536	
$\sin Q,\ \ \sin Q_0$	+0·38570	+0·38283	−0·38061
$\cos Q,\ \ \cos Q_0$	+0·92264	+0·92383	−0·92475
$\mp p \cos Q_0 \pm q_0 \sin Q_0,\ \ a'$	− 62	− 73	− 32
Sum	− 135		− 105
$\mp$ Sum $\div \sqrt{a^2 + b'^2} = \Delta Q$	+ 311		− 240
$\Delta Q\ .\ \cos Q_0 = \Delta \sin Q$	+ 287		− 222
$-\Delta Q\ .\ \sin Q_0 = \Delta \cos Q$	− 119		+ 92
$a' - p \cos Q + q \sin Q$	+ 2		− 3
$\cos \phi_1 \cos \theta$	+0·59374	+0·58849	+0·58274
$\sin \phi_1$	−0·49210	−0·50565	−0·51918
Sum of squares − 1	0	0	+ 1
$\tan \theta$ or $\cot \theta$	+0·93262	+0·93282	+0·93209
$\cos \theta$ or $\sin \theta$	+0·73131	+0·73124	+0·73151
μ		+32·589	
θ	+46·997	+46·991	+47·013
$\tan \phi$	−0·56719	−0·58808	−0·60952

A very effective independent check on the Besselian elements, etc., which provides against the possibility of systematic error, is obtained by computing, for some selected time, the apparent right ascension and declination of the Sun and Moon, as seen from the places reputed to be on the central line and limits respectively. For the place on the central line, these quantities should be equal. For the places on the limits, the difference between the centres of the Sun and Moon should be equal to the difference of their semi-diameters, allowance being made for augmentation ; further, the difference of semi-diameters, multiplied by $\sin Q$ and $\cos Q$, should yield the differences in right ascension and declination respectively. To check the duration, the apparent positions should also be computed, in the case of the point on the central line, for times 10^m before and after the given G.M.T., in order to deduce the rate of apparent motion of the Moon relative to the Sun ; the duration is equal to the difference of semi-diameters divided by this rate. The formulæ required are, using accents for apparent positions :—

For Moon :

$$a = \frac{\sin \pi \rho \cos \phi'}{\cos \delta} \qquad \tan \Delta\alpha = \frac{a \sin h}{1 - a \cos h} \qquad \alpha' = \alpha - \Delta\alpha$$

$$\tan \delta' = \left(1 - \frac{\sin \pi \rho \sin \phi'}{\sin \delta}\right) \frac{\sin (h + \Delta\alpha)}{\sin h} \tan \delta$$

where $\tan \delta$ is taken to 7 decimals. The ratio $\dfrac{\sin (h+\Delta\alpha)}{\sin h}$ is not affected by a slight error in h.

For Sun :

$$\Delta\alpha^s = \frac{8{\cdot}80}{15R} \rho \cos \phi' \sin h \sec \delta \qquad \alpha' = \alpha - \Delta\alpha$$

$$\tan \gamma = \frac{\tan \phi'}{\cos h} = \frac{0{\cdot}9933 \tan \phi}{\cos h}$$

$$\Delta\delta = \frac{8{\cdot}80}{R} \rho \sin \phi' \frac{\sin (\gamma - \delta)}{\sin \gamma} \qquad \delta' = \delta - \Delta\delta$$

For augmentation :

s = geocentric S.D.
s' = apparent S.D.

$$\tan \gamma = \frac{\tan \phi' \cos \tfrac{1}{2}\Delta\alpha}{\cos (h + \tfrac{1}{2}\Delta\alpha)}$$

$$s' = s \frac{\sin (\gamma - \delta')}{\sin (\gamma - \delta)}$$

It is usually necessary to apply some corrections to the tabular positions of the Sun and Moon. Corrections to the true longitude and latitude are converted to corrections in right ascension and declination by

$$\cos \delta \,.\, \Delta\alpha = \cos S \cos \beta \,.\, \Delta\lambda - \sin S \,.\, \Delta\beta$$
$$\Delta\delta = \sin S \cos \beta \,.\, \Delta\lambda + \cos S \,.\, \Delta\beta$$

where $\sin S = \cos\lambda \sec\delta \sin\epsilon = \cos\alpha \sec\beta \sin\epsilon$ and S lies between $\pm 90°$. More frequently the corrections are to the mean longitudes, in which case they become

For the Sun	$0{\cdot}006764\Delta\lambda \times$ Var. per hour
For the Moon	$0{\cdot}03036\ \Delta\lambda \times$ Var. per minute

Although the above corrections are usually applied by the computer before elements are published in the almanacs, it not infrequently happens that a later revision is desired. The only elements that require alteration are x and y, to which the corrections to be applied are

$$\Delta x = \frac{15\cos\delta_{\leftmoon}(\Delta\alpha_{\leftmoon} - \Delta\alpha_{\odot})}{\pi_{\leftmoon}}$$

$$\Delta y = \frac{\Delta\delta_{\leftmoon} - \Delta\delta_{\odot}}{\pi_{\leftmoon}}$$

where $\pi_{\leftmoon}$ is the Moon's horizontal parallax in seconds of arc. It would be helpful if the ephemerides would publish, for each eclipse, the coefficients a, b, c, d in

$$\Delta x = a\,.\,\Delta\lambda_{\odot} + b\,.\,\Delta\lambda_{\leftmoon}$$
$$\Delta y = c\,.\,\Delta\lambda_{\odot} + d\,.\,\Delta\lambda_{\leftmoon}$$

in which $\Delta\lambda_{\odot}$ and $\Delta\lambda_{\leftmoon}$ are the *supplementary* corrections that the revising computer wishes to apply to the mean longitudes.

In the computation of local times of beginning and ending of eclipse, it will be found better, instead of computing ξ', η', n, N, ψ, etc., to compute $m = \sqrt{(x-\xi)^2 + (y-\eta)^2}$ for two or three integral minutes of G.M.T. in the neighbourhood of the contact required, and then interpolate to the time when $m - L = 0$. The position angle P of the point of contact is given by

$$\sin P = x - \xi \qquad \cos P = y - \eta$$

so that P is the value of M at the moment of contact.

Simple formulæ for the shape of the Moon's shadow on the Earth at a given moment have been given by the writer in an earlier paper.*

THE TOTAL SOLAR ECLIPSE OF 1940 OCTOBER 1.

L. J. Comrie.

There are no scientifically observable total solar eclipses in 1935, 1938 and 1939. The eclipse of 1936 June 19 can be seen favourably in Southern Siberia, a locality that is hardly likely to attract any but the most ardent

* " Some Computational Problems arising in Eclipses," *M.N.*, **87**, 483, 1927.

eclipse observers. The eclipse of 1937 June 8, with the exceptionally long duration of seven minutes, passes over the Pacific Ocean, apparently without touching any land whatsoever. It seems, therefore, that a certain interest should attach to the eclipse of 1940 October 1, as being the next that is likely to attract extended astronomical interest. The eclipse of 1941 September 21 may be very favourably observed from the east coast of China, after which the astronomer must wait till 1944 January 25, for an eclipse which crosses Brazil.

The positions of the Sun have been taken from Newcomb's *Tables*, with the addition of a correction of $+1''\cdot6$ to the mean longitude, and those of the Moon from Brown's *Tables*, with the addition of a correction of $+4''\cdot0$ to the mean longitude and $-0''\cdot5$ to the true latitude. These corrections reduce to

Sun	$\Delta\alpha = +0^s\cdot10$	$\Delta\delta = -0''\cdot6$
Moon	$\Delta\alpha = +0^s\cdot28$	$\Delta\delta = -1''\cdot9$

The Besselian elements for the portions of the track that are of immediate interest are shown in the table below.

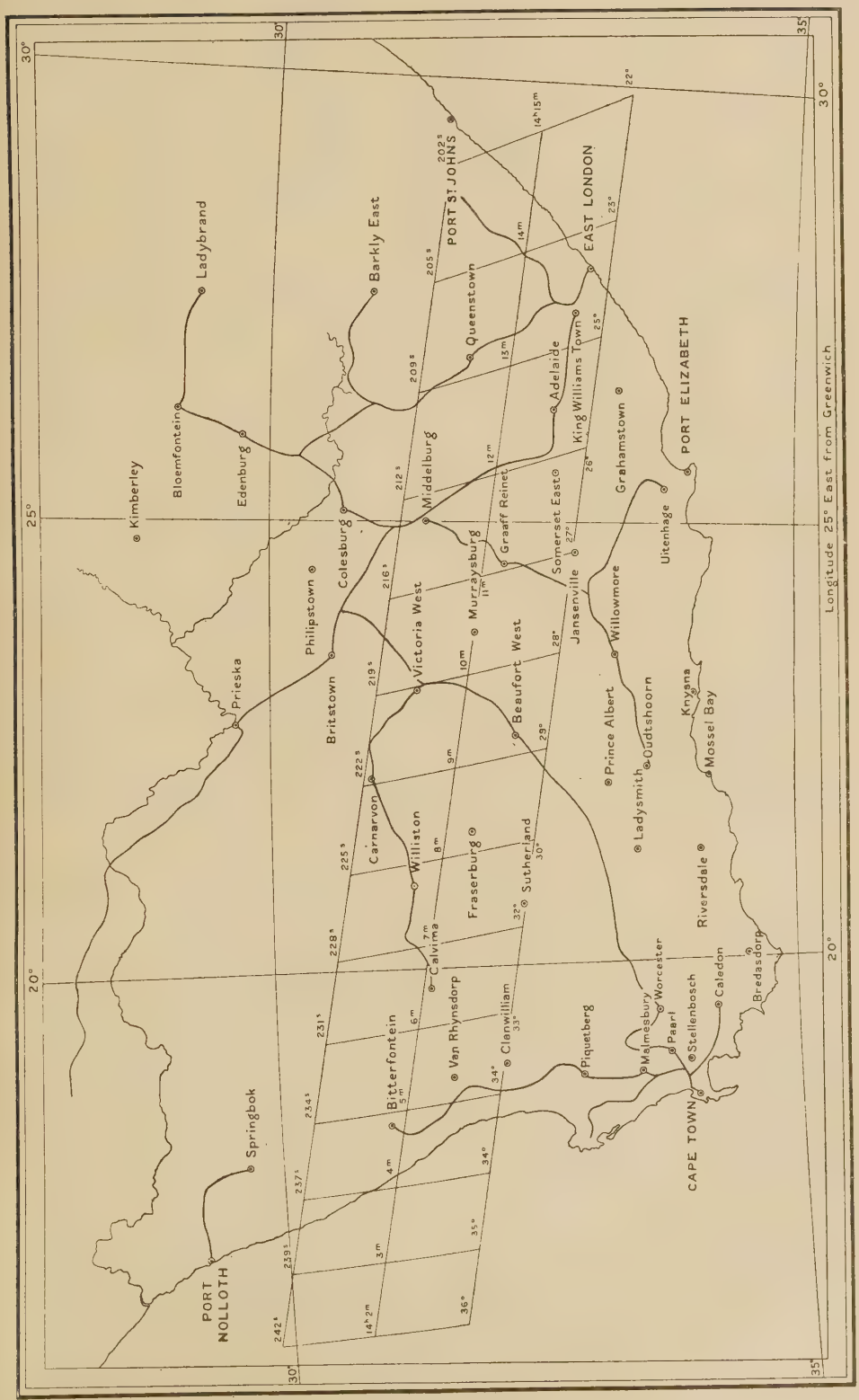
G.M.T.	x	y	sin d	cos d	μ	l_2
h m					°	
11 30	−0·76016	−0·02919	−0·05626	+0·99841	− 4·922	−0·01205
11 40	·66742	·05857	·05631	·99841	− 2·421	·01205
11 50	·57468	·08794	·05636	·99841	+ 0·079	·01205
12 00	−0·48194	−0·11731	−0·05640	+0·99841	+ 2·580	−0·01204
...	...	...	...	...	...	...
14 00	+0·63087	−0·46962	−0·05695	+0·99838	+32·589	−0·01207
14 10	·72360	·49896	·05700	·99837	35·090	·01208
14 20	+0·81633	−0·52831	−0·05704	+0·99837	+37·591	−0·01208

$\tan f_2 = 0\cdot00465$

The portion of the central line that lies in Eastern Brazil is as follows:—

G.M.T.	ϕ	λ	Duration	Altitude
h m	° ′	° ′	s	°
11 30	−3 47	+44 42	252	40
11 35	4 48	42 05	263	44
11 40	5 48	39 43	273	48
11 45	6 46	37 32	283	51
11 50	7 44	35 31	291	54
11 55	8 41	33 38	299	57
12 00	−9 37	+31 50	306	60

The part of the track that crosses South Africa is shown in the following table and also in an accompanying map, in which the G.M.T. of mid-totality is shown along the central line, the duration on the central line is shown on the northern limit, and the altitude on the central line is shown on the southern limit. The lines crossing the track are lines of equal mid-totality. The track is 120 miles wide.


Ladybrand
Barkly East
PORT ST JOHNS
EAST LONDON
Queenstown
Adelaide
King Williams Town
Bloemfontein
Edenburg
Middelburg
Graaff Reinet
Somerset East
Grahamstown
PORT ELIZABETH
Kimberley
Colesburg
Murraysburg
Jansenville
Uitenhage
Philipstown
Victoria West
Willowmore
Prieska
Britstown
Beaufort West
Prince Albert
Oudtshoorn
Knysna
Mossel Bay
Carnarvon
Williston
Fraserburg
Sutherland
Ladysmith
Riversdale
Calvinia
Van Rhynsdorp
Clanwilliam
Worcester
Malmesbury
Paarl
Stellenbosch
Caledon
Bredasdorp
Piquetberg
Bitterfontein
Springbok
CAPE TOWN
PORT NOLLOTH
Longitude 25° East from Greenwich

G.M.T.	Northern Limit of Totality		Central Line		Southern Limit of Totality		Duration on Central Line	Altitude on Central Line
	ϕ	λ	ϕ	λ	ϕ	λ		
h m	° ′	° ′	° ′	° ′	° ′	° ′	s	°
14 00	−29 34	−14 24	−30 28	−14 24	−31 22	−14 25	247	38
01	29 43	15 09	30 37	15 10	31 31	15 12	244	37
02	29 52	15 55	30 46	15 57	31 40	16 00	242	36
03	30 01	16 42	30 54	16 45	31 48	16 50	239	35
04	30 10	17 31	31 03	17 35	31 57	17 41	237	34
14 05	−30 18	−18 21	−31 12	−18 26	−32 06	−18 34	234	34
06	30 27	19 13	31 20	19 20	32 14	19 29	231	33
07	30 36	20 07	31 29	20 16	32 22	20 26	228	32
08	30 44	21 04	31 37	21 13	32 30	21 25	225	30
09	30 53	22 03	31 45	22 14	32 38	22 27	222	29
14 10	−31 01	−23 04	−31 53	−23 17	−32 46	−23 32	219	28
11	31 09	24 08	32 01	24 23	32 54	24 40	216	27
12	31 17	25 15	32 09	25 32	33 01	25 52	212	26
13	31 24	26 27	32 16	26 46	33 08	27 08	209	25
14	31 32	27 42	32 23	28 04	33 15	28 29	205	23
14 15	−31 39	−29 02	−32 30	−29 27	−33 21	−29 56	202	22
16	31 46	30 29	32 37	30 57	33 27	31 29	198	21
14 17	−31 52	−32 03	−32 43	−32 35	−33 33	−33 12	193	19

The above tables have been computed rigorously and any subsequent changes in the adopted corrections to the Sun and Moon will not appreciably affect the tracks given, or the durations, although the times of beginning and ending of totality will be changed approximately 2^s for every $1''$ change in the relative positions of the Sun and Moon. Further changes in the mean longitudes of $\Delta\lambda_\odot$ and $\Delta\lambda_\leftmoon$ (in seconds of arc) yield, for this eclipse,

$$\Delta x = +0{\cdot}00030\,\Delta\lambda_\leftmoon - 0{\cdot}00025\,\Delta\lambda_\odot$$
$$\Delta y = -0{\cdot}00010\,\Delta\lambda_\leftmoon + 0{\cdot}00011\,\Delta\lambda_\odot$$

from which, with the remaining Besselian elements unchanged, the times of beginning and ending may be computed in the usual way.

In South Africa the best astronomical conditions occur on the west coast,* about 200 miles north of Cape Town, where the altitude is about 33° and the duration just under 4^m. At Pernambuco, on the east coast of Brazil, the altitude is 20° greater and the duration 1^m longer.

* Farther inland the greater height above sea-level compensates, to some extent, for the lower altitude of the Sun.

ON A NEW AIRCRAFT-SEXTANT FOR USE WITH VISIBLE HORIZON.

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1. When this instrument is in its observing position the visible horizon is reflected at three points vertically downwards into a telescope. The portions of the horizon observed are facing the observer and lie 60° to his right and left. The images of the visible horizon appear as three straight lines in the field of view intersecting at about 60°. The observation consists in bringing the image of the celestial object into a certain point defined by the configuration of the horizon-lines. It is of no consequence whether all the rays after the downward reflection are again reflected into a horizontal direction, and whether this occurs before or after the rays enter the object-glass. It will be shown below how the observer is enabled to keep the error of inclination so small that its effect on the result, of second order, can be neglected.

2. *Description of Instrument.*—The frame of the instrument, similar to that of a sextant, supports a right-angled prism as index-mirror whose position can be read on a divided circle. The telescope points to the index-prism and may be detachable. A glass plate is firmly fixed on the frame between object-glass and index-prism at about right angles to the telescope, and carries, cemented to it, three reflecting prisms 60° apart from each other. They are right angled, the angle at the plate not being quite 45°, and are so arranged that light propagated parallel to the plate towards the prisms is reflected into the telescope. For convenience of observing, the eye-end of the telescope may be at right angles to the optical axis. In accordance with § 1 a right-angled prism may be substituted for the plate, and the telescope may lie in a horizontal direction when the instrument is in the observing position (fig. 5).

3. *Theory.*—Let all the directions entering the discussion be displaced parallel to themselves to the centre of a sphere so that the intersections with the sphere represent the directions. In the diagram, A_1A_2 represents the plane of the glass plate, which serves as reference plane. Its pole, M, would stand for the optical axis of the telescope if this were perpendicular to the glass plate. Q_1 on MA_1 stands for the normal to the middle mirror, and Q_2 on MA_2 for the normal to the mirror facing the horizon towards the left. The angle enclosed between the edges of these mirrors, here about 60°, is ϕ_2, and is represented by the arc A_1A_2. The arcs MQ_1 and MQ_2 are respectively $45° - c_1$ and $45° - c_2$, and designate the inclinations of these mirrors to the glass plate. Similarly, the pole Q_3 of the third mirror is situated behind the plane of drawing, and its position is given by ϕ_3 and c_3.

I assume that the instrument has been adjusted in the same manner as a sextant, the middle mirror replacing the horizon-glass ; then the plane of the circle is also represented by the great circle MA_1, and the normal, I, to the index-glass moves on this great circle. Let R_0 be the circle-reading when I lies in Q_1, *i.e.* when there is coincidence of the two images of a distant object due to rays reflected by index-glass and middle mirror (as in the ordinary sextant). Let an object, S, be taken under observation while the plane of the circle is held vertically, whereby both S and the zenith, Z, lie on the great circle MQ_1I. Errors arising from an inclined position of the plane of the instrument, or from maladjustment, have the same effect as with an ordinary sextant and need not be considered here. With these reservations, the astronomical horizon, H_1H_2, intersects the reference great circle, A_1A_2, at a point 90° from A_1 and H_1. Hence $A_1H_1 = i$, $A_2H_2 = i \cos \phi_2$, if i designates the inclination of the glass plate to the astronomical horizon. The visible horizon is represented by a small circle, K_1K_2, parallel to H_1H_2, at angular distance D, the dip of the horizon.

The astronomical and visible horizons intersect the great circle MQ_1 at right angles in H_1 and K_1 respectively. The images of the points A_1, H_1, K_1 as reflected by Q_1 lie respectively equidistant from Q_1, and their relative positions are shown in fig. 2 on the assumption that D is less than $2c_1$. In this diagram $MA_1' = 2c_1$, $A_1'H_1' = i$, $H_1'K_1' = D$, and the visible horizon intersects MH_1' in K_1' at right angles.

Similarly, the astronomical and visible horizons intersect the great circle MQ_2 at H_2 and K_2 at an angle $(90° - i \sin \phi_2)$. The images of the points A_2, H_2, K_2 as reflected by Q_2 are designated by dashes and marked in the diagram. $MA_2' = 2c_2$, $A_2'H_2' = i \cos \phi_2$, $H_2'K_2' = D$, and the image of the visible horizon intersects MK_2' in K_2' at an angle $(90° - i \sin \phi_2)$.

The images reflected by the third mirror are shown with suffix 3 in the figure.

If the eye-piece were placed in the optical axis of the telescope, fig. 2 would give the relative positions of the images with M as centre of the field of view. Nothing is changed in this configuration by a reflection common to all the rays, *e.g.* near the eye-end in a periscopic telescope, or, before entering the telescope, by a right-angled prism whose hypotenuse lies about parallel to the middle prism. Only three lines of the visible horizon are perceptible. The left and right horizon lines intersect at a point designated by X.

(*a*) Let the image of the object, S, be brought into coincidence with point X by turning the normal, I, to the index-prism from Q_1 to I_x, in which position the reading is R_x ; then $Q_1I_x = \frac{1}{2}(R_x - R_0)$, provided that the figures on the circle are doubled. The astronomical altitude of S is $h = SH_1 = SI_x - H_1I_x$. Substitute $H_1I_x = H_1Q_1 - I_xQ_1$, and $H_1Q_1 = Q_1H_1'$, $SI_x = I_xX$, and obtain

$$h = R_x - R_0 + \delta R_x, \quad \text{where} \quad \delta R_x = XH_1'.$$

To find XH_1', measure $XK_1 = s$ (see below), then $XH_1' = s + D$. Further, $XH_1' = MH_1' - MX$ and $MH_1' = i + 2c_1$; hence $+2c_1 - s - D = MX - i$.

From this I obtain an expression for D in terms of s, i, c and ϕ in the following way.

Let $K_2'X$ intersect MH_1' at X_2. MX_2 in triangle $MK_2'X_2$ may be calculated from $MK_2' = 2c_2 + i\cos\phi_2 - D$, the angle ϕ_2 at M, and the angle $(90° + i\sin\phi_2)$ at K_2'. In the same way let $K_3'X$ intersect MH_1' at X_3, then MX_3 may be calculated in an analogous way. Both combined give

$$MX - i = \frac{2c_2\sin\phi_3 + 2c_3\sin\phi_2 - D(\sin\phi_2 + \sin\phi_3)}{\sin(\phi_2+\phi_3)}.$$

The second-order terms which contain i^2, iD and ic have been omitted. Note that the expression for D in terms of s, c and i (which is not put down here) does not contain the first power of the inclination i.

$$\delta R_x = \frac{s(\sin\phi_2+\sin\phi_3) - 2c_1\,.\,\sin(\phi_2+\phi_3) + 2c_2\sin\phi_3 + 2c_3\sin\phi_2}{\sin\phi_2+\sin\phi_3-\sin(\phi_2+\phi_3)}.$$

It is feasible to mount the three prisms with an accuracy of a degree and to make $\phi_2 = \phi_3 = \phi = 60°$, also to give the same base angle to the three prisms, $c_1 = c_2 = c_3 = c$, in which case $\delta R_x = 2s + 2c$. s is measured by means of a prism of refracting angle of about five degrees, which is mounted in front of the middle mirror (fig. 5) and capable of rotation round the normal facing the observer placed in the direction of A_1 (fig. 1). The value of s gives also the height of the observer above sea-level, since the dip of the horizon is, as stated above, a function of s and c.

(*b*) In a second method the image of S is brought to a point, N, situated at equal distances, ρ, from the images of the three visible horizon lines. At this position of the index-mirror the normal to the index-prism lies at I_n and the reading is R_n. In this case

$$h = R_n - R_0 + \delta R_n,$$

where $\delta R_n = NH_1' = D + \rho$.

The perpendicular from M to $K_2'X_2$ differs from MK_2' by quantities of third order in i, c and D which are neglected.

Let $MN(=r)$ enclose angle θ with MH_1'; then

$$\begin{aligned}\rho &= MK_1' - r\cos\theta\\ \rho &= MK_2' - r\cos(\phi_2+\theta-i_2)\\ \rho &= MK_3' - r\cos(\phi_3-\theta-i_3)\end{aligned}$$

where

$$i_2 = i\sin\phi_2, \quad \text{and} \quad i_3 = i\sin\phi_3.$$

Further,

$$\begin{aligned}MK_1' &= 2c_1 - D + i\\ MK_2' &= 2c_2 - D + i\cos\phi_2\\ MK_3' &= 2c_3 - D + i\cos\phi_3.\end{aligned}$$

$(\rho + D)$ and $r\sin\theta$ can be eliminated from the three equations, thus giving

$$r\cos\theta = i + \frac{2(c_1-c_2)\sin\phi_3 - 2(c_1-c_3)\sin\phi_2}{\sin\phi_2+\sin\phi_3-\sin(\phi_2+\phi_3)}.$$

The second-order terms which have been omitted contain i^2 and ic. Apart from these terms, the substitution gives

$$\delta R_n = \frac{-2c_1 \sin(\phi_2+\phi_3)+2c_2 \sin\phi_3+2c_3 \sin\phi_2}{\sin\phi_2+\sin\phi_3-\sin(\phi_2+\phi_3)}.$$

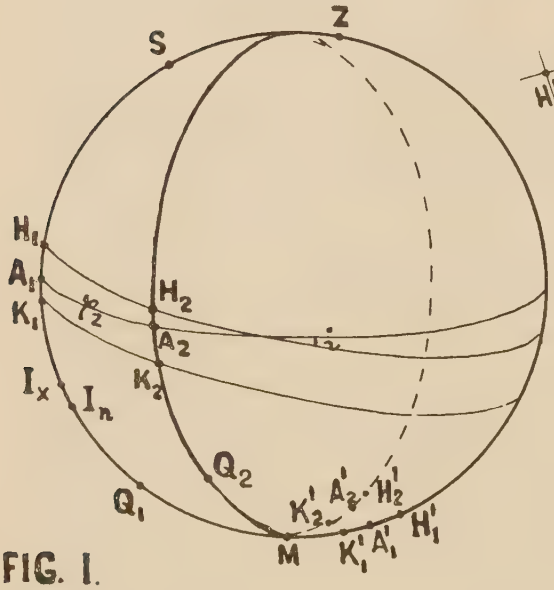


FIG. 1.

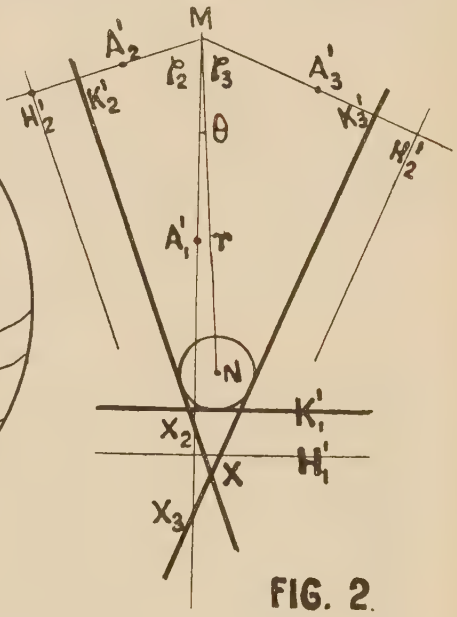


FIG. 2.

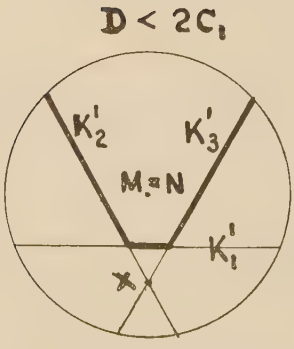


FIG. 3.

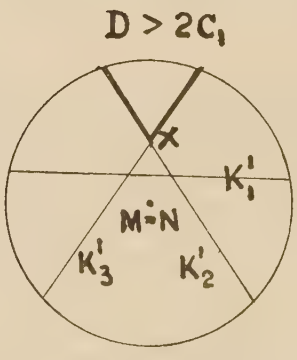


FIG. 4.

Here again the first power of the inclination, i, does not enter the correction to the index-error. In the case cited, where $\phi = 60°$, and the inclinations of the three mirrors are all equal to $(45° - c)$, θ becomes zero and $\delta R_n = 2c$.

In a set of three small prisms, 8 mm. on a side, mounted on a glass plate and supplied by Messrs. Adam Hilger, Ltd., London, $c_1 = 0$, $c_2 = +12'$, $c_3 = +6'$, $\phi_2 = 55° 28'$, $\phi_3 = 61° 45'$, wherewith $\delta R_n = +38'$.

4. *Most Favourable Observing Condition.*—According to the above, all the second-order terms contain i. This error is the inclination at which the observer holds the instrument, and it is, therefore, important that the observer has some means of ascertaining whether he holds his instrument correctly. The error i is, for equal values of c (and the optician is aiming at that), equal to r, and hence equal to MN. But M is the centre of the field, so that i becomes small when the point N, equidistant from the images

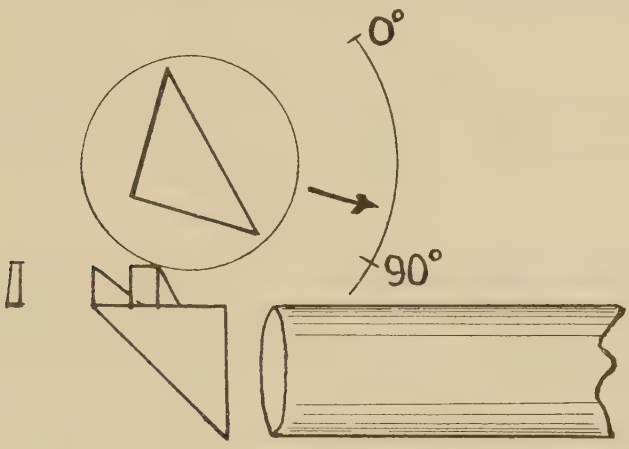


FIG. 5a

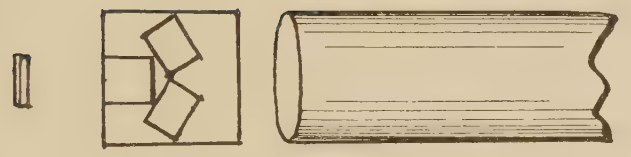


FIG. 5b

of the three visible horizons, is placed near the centre of the field. This need only be done approximately, say to $\frac{1}{2}°$, since in absolute measure a degree squared is equivalent to only a minute.

Figs. 3 and 4 show the appearance in the field of view when N is placed at the centre of the field. For equal values of c, the middle horizon bisects the distance MX, and $XK_1' = s = D - 2c$. The astronomical horizon, H_1', lies in the field below the visible horizon, K_1', and correspondingly for K_2' and K_3'. The light from the sky extends in the direction K to H, so that in the two cases shown in the diagrams, in which $D < 2c$, and $D > 2c$, the dark portion of the field is limited by the bold lines. For intense illumination of the sky the intersection, X, may be invisible for $D < 2c$, and the front horizon-line may be invisible for $D > 2c$. It thus appears that

at daytime the second (*b*) mode of observing is preferable to that given under (*a*), provided that $D < 2c$. If $c = 1°$, *i.e.* the angle of inclination of the three mirrors to their base is 44°, the dip of the horizon would be smaller than $2c$ up to 5000 metres above sea-level, and for $c = 1\frac{1}{2}°$ up to 10,000 metres. An inclination of about 44° appears to be the best compromise.

In most cases where both the front horizon and the intersection, X, of the other two horizons are perceptible the method (*a*) will be more accurate.

OBSERVATIONS OF LEONIDS AT HELWAN OBSERVATORY, 1932 NOVEMBER.

P. A. Curry.

At the request of the Astronomer Royal a careful watch was kept for the return of the Leonids, and arrangements were made, in the event of a large shower being observed early in the night, to telegraph to the British Broadcasting Corporation, who agreed to announce the fact during their programme.

No systematic observations of meteors had been made here before, and careful preparations were made to deal with a large shower. A special star chart was made and mounted on ground glass. The positions of stars down to the fourth magnitude were marked by small punched holes varying in size according to the magnitude of the stars and illuminated from below by means of a small four-volt lamp. The chart was sprayed a matt black surface underneath, and was mounted on the ground glass in a wooden sloping frame. This is the method described by Mr. A. R. Hinks in the *M.N.* for 1900 April, and was found a great help in plotting the trails rapidly. The night of the 15th was partially clouded, a magnificent lunar halo persisting all night. The chart was tested that night and it was found that the punctured holes were too big. No meteors were observed from 11.30 p.m. until 12.30 p.m.

A fresh chart was prepared with smaller holes and this proved very satisfactory.

On the 16th all was in readiness on the roof of the observatory. The writer, whose duty was to plot the trails, was assisted by Dr. Rida Eff. Madwar, the resident astronomer ; Abdel Hamid Eff. Samaha, assistant astronomer ; Abdel Hafiz Eff. Ahmed, computer ; and three of the observatory servants, one of whom, a keen-sighted Bedouin, missed very little.

One assistant kept a written record of each meteor observed, another tapped a key which recorded on the chronograph, a third used an adding machine and the Bedouin counted by dropping pebbles in a bag.

There was a heavy bank of alto-cumulus cloud drifting across the sky, with bright patches of sky visible and a light higher patch of cirro-stratus in which a very fine lunar halo was outlined most of the night. There was

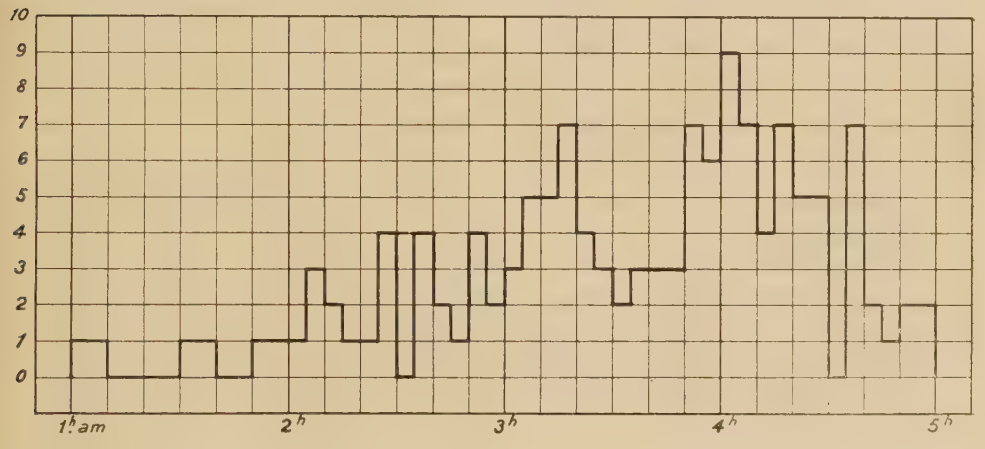
a little haze for about 10° altitude, which obscured 3rd and 4th magnitude stars when there was no cloud.

Throughout the whole night the sky was on the average about half clouded.

133 meteors were observed, nearly all appearing to radiate from Leo. The first one observed was at $1^h\ 3^m$, the hourly counts being 1^h–2^h, 6; 2^h–3^h, 25; 3^h–4^h, 51; 4^h–5^h, 51.

The clouds increased before sunrise, and at 5 a.m. the sky was three-quarters clouded and the observations ceased. Estimates of the brightness were made in nearly all cases, 47 being considered brighter than Jupiter, 62 between Jupiter and Regulus, and 17 fainter than Regulus. Of the 47 bright meteors, 16 were particularly bright, one of them observed at

Number of Meteors Observed in Five-Minute Intervals
1932 Nov. 17th



$4^h\ 44^m$ being considered as bright as Venus and leaving a greenish trail which persisted for 10 seconds.

The diagram shows the numbers observed in 5-minute intervals, the maximum 5-minute count being 9 from 4^h to $4^h\ 5^m$.

The weather on the night of the 17th looked much more promising, there being very little cloud, but a thick haze was found to extend from the horizon to an altitude of 10° to 15° through which even Jupiter was only visible with difficulty.

The same arrangements were made as before and a careful look-out was kept from 11 p.m. The first meteor was seen at $12^h\ 40^m$ and continuous observation was kept until 5 a.m. Heavy cloud developed about 3.30 a.m. and by 4.30 the sky was three-quarters clouded. In all, 49 meteors were seen, nearly all appearing to radiate from Leo; 13 of these were estimated to be brighter than Jupiter and 17 between Jupiter and Regulus, and the remainder less than Regulus.

The hourly counts were: 12^h–1^h, 2; 1^h–2^h, 8; 2^h–3^h, 13; 3^h–4^h, 9; 4^h–5^h, 17.

One, observed at $4^h\ 05^m$, was particularly bright and left a trail lasting about 2 seconds.

On the night of the 18th only 7 meteors were seen up till 3 a.m., when the sky was completely clouded in the east.

The time of maximum display during the hours of observation appears to have been about 4 a.m. on November 17. It was only possible to see meteors down to about 2nd or 3rd magnitude owing to the moonlight and the clouds and haze, and it is quite possible that the number seen would have been very much greater if meteorological conditions had been more favourable. It is, however, evident that there was no return of the magnificent swarms of Leonids seen in 1833 and 1866.

On the morning of the 17th the trails of 33 meteors were plotted. The radiant point was determined graphically, no more refined method of determination being applied as the observations did not seem to warrant it. Two independent observers chose pairs of paths as nearly as possible at right angles, the first 12 pairs and the second 10, the mean positions of the radiant points found being

R.A.	Dec.
$153°{\cdot}7 \pm 0°{\cdot}7$	$22°{\cdot}9 \pm 0°{\cdot}7$ N.
$152°{\cdot}9 \pm 0°{\cdot}6$	$22°{\cdot}3 \pm 0°{\cdot}8$

The mean of these probably gives as good a position of the radiant as the observations justified.

Mean radiant point R.A. $153°{\cdot}3$, Dec. $22°{\cdot}6$ N.

On the morning of the 18th 10 trails were plotted, and these with rather poor angles of intersection gave a radiant point R.A. $155°{\cdot}1$, Dec. $26°{\cdot}0$ N. No great reliance can be placed on this as the scatter was considerable.

The times given are all Egyptian Civil Time, which is 2 hours fast on Greenwich Mean Time.

MICROMETRICAL MEASURES OF DOUBLE STARS.

(24TH SERIES.)

T. E. Espin and W. Milburn.

The micrometrical measures are given as in previous lists, those marked E being made with the 24-inch, and the ones marked M with the $17\frac{1}{4}$-inch.

Name	1900 R.A.	1900 Decl.	P.A.	Dist.	Mags.	Nights	Date 1932 +	
	h m	° ′	°	″				
Es 1941	0 29·3	+37 44	338·6	2·56	9·1 9·2	1	·019	Es
			342·9	2·54	...	1	·033	M
Es 2449	1 21·3	37 11	177·7	6·43	10·8 11·2	3	·067	Es
Es 159	45·1	37 17	17·5	8·32	8·6 12·0	2	·073	Es

Name	1900 R.A.	1900 Decl.	P.A.	Dist.	Mags.	Nights	Date 1932 +		
	h m	° ′	°	″					
Σ 179	1 47·3	+36 50	161·3	3·12	8·3 8·6	2	·067		Es
Es 2454	2 43·6	37 59	172·7	7·04	9·7 10·2	3	·073		Es
Btn 131	3 2·7	27 35	243·8	3·60	10·0 10·0	1	·107		M
Es 274	22·1	35 37	142·7	3·18	9·7 9·7	3	·073		Es
Es 276	37·3	37 40	288·0	7·41	8·3 13·0	3	·135		Es
OΣ 78	4 9·8	29 47	249·3	2·37	...	2	·130		M
Σ 529	16·7	28 10	14·9	4·02	...	2	·130		M
Σ 577	35·6	37 19	47·7	1·74	7·2 7·3	3	·163		Es
A 1845	59·0	27 31	138·5	1·87	9·5 9·6	2	·130		M
Σ 1421	10 12·4	28 1	332·6	4·75	7·5 8·5	1	·279		M
Es 1639	14·8	37 49	134·4	2·62	9·5 10·2	1	·279		Es
Es 1737	41·4	36 10	281·7	2·16	9·5 9·6	1	·279		Es
Es 469	17 42·8	28 1	151·4	3·90	10·0 12·0	2	·609	BC	M
			268·9	...	...	1	·564	AB	M
h 1321	18 18·0	39 17	82·2	9·18	...	2	·739		M
Es 475	24·0	27 4	224·1	9·46	...	2	·574		M
Sch 588	19 14·8	38 59	82·3	1·62	10·0 12·0	2	·669		M
Sch 632	30·0	38 32	253·7	7·16	10·0 10·3	1	·583		Es
			255·4	7·16	...	1	·695		M
Es 1984	20 51·6	38 48	116·3	3·04	10·0 11·5	2	·731		M
Es 261	21 48·2	39 57	166·1	4·51	9·5 9·5	2	·731		M

Espin No.	B.D.	1900 R.A.	1900 Decl.	P.A.	Dist.	Mags.	Nights	Date 1932 +	
	°	h m	° ′	°	″				
2544*	...	0 8·9	+39 23	90·4	6·43	10·0 11·5	2	·446	
2545	+36·73	25·1	37 10	246·9	10·92	9·5 10·0	2	·018	
2546	...	29·6	37 29	185·4	6·12	10·8 11·0	2	·018	
2547*	...	1 7·9	30 25	57·2	4·89	11·0 12·0	2	·457	
2548	...	21·3	37 9	167·7	6·81	11·5 12·0	2	·041	
2549	...	29·2	36 36	202·1	5·68	11·0 11·1	2	·067	
2550	...	42·8	36 46	107·2	3·34	11·0 11·0	2	·069	
2551*	...	2 19·1	36 40	22·6	4·81	11·0 11·7	3	·086	
2552	...	43·5	36 39	249·7	4·02	10·7 10·9	2	·077	
2553	...	43·5	36 56	168·9	6·63	10·5 12·0	2	·067	
2554	+37·650	48·0	37 29	159·5	6·71	9·3 11·0	3	·068	
2555	...	48·1	36 18	179·4	6·18	10·2 10·3	2	·093	
2556	...	3 6·4	36 54	202·0	7·24	10·0 10·1	2	·073	
2557	...	10·7	36 26	62·6	3·08	10·7 11·0	3	·122	
2558	+35·673	15·6	35 17	257·3	6·48	9·0 13·0	2	·067	
2559	+35·709	25·6	35 14	277·9	4·35	9·5 10·7	2	·075	
2560	...	37·7	36 51	316·3	7·90	10·5 12·0	2	·106	
2561	+36·814	58·4	36 28	57·5	6·48	8·5 12·0	2	·139	
2562*	+37·944	4 29·0	37 48	43·2	6·25	11·0 11·4	2	·100	BC
2563	+36·1185	5 28·4	36 50	138·8	3·62	10·0 12·5	1	·183	
2564	...	6 19·4	38 5	262·6	5·08	10·0 11·5	1	·071	
2565	...	9 10·7	36 5	285·6	10·04	10·5 11·0	1	·279	
2566	...	10 13·3	36 50	37·6	4·58	11·0 11·1	1	·279	
2567*	...	17 46·2	37 58	196·6	6·16	11·0 11·0	1	·583	
2568	...	48·4	37 57	36·6	9·27	9·7 10·0	2	·576	
2569*	+39·3341	18 7·4	39 2	273·7	9·30	9·5 10·0	2	·572	

Espin No.	B.D.	1900 R.A.	1900 Decl.	P.A.	Dist.	Mags.		Nights	Date 1932 +
	°	h m	° ′	°	″				
2570*	...	18 15·6	+39 15	313·6	6·79	10·5	10·5	1	·583
2571*	...	32·9	38 53	71·8	6·03	10·5	10·7	2	·577
2572	...	44·6	38 51	66·6	6·59	10·0	11·5	2	·577
2573*	...	54·4	38 44	22·0	5·29	11·0	11·5	2	·577
2574*	...	19 25·0	38 25	96·7	5·50	11·0	11·5	1	·583
2575*	...	42·3	39 21	188·3	3·91	10·6	10·6	1	·564

Notes on stars marked *

Nos. 2544 and 2547. Measured in 1931.
2551. *Comes* to B.D. +36·478 at P. 315°·5.
2562. AB; 354°·7; 30″·2 : 1932·071.
2567. M. 1932·68. P. 193°·4, D. 5·85, 3nts. *Comes* 10·7, P. 260°·7, 25″ ±.
2569. M. 1932·68. P. 275°·2, D. 9″·28, 2 nts.
2570. M. 1932·69. P. 312°·2, D. 6″·00, 2 nts.
2571. M. 1932·64. P. 72°·6, D. 6″·04, 1 nt.
2573. M. 1932·68. P. 20°·6, D. 4″·31. Mags. 11·0, 12·4, 2 nts.
2574. M. 1932·64. P. 95°·3, D. 4″·79, 1 nt.
2575. M. 1932·64. P. 189°·2, D. 3″·29. Mags. 9·5, 9·6, 1 nt.

Milburn No.	B.D.	1900 R.A.	1900 Decl.	P.A.	Dist.	Mags.		Nights	Date 1932 +	
	°	h m	° ′	°	″					
733	+39·53	0 13·2	+39 51	55·4	9·52	9·0	10·5	2	·856	
734	...	36·2	39 44	181·7	6·73	10·0	11·5	1	·756	
735*	...	56·0	25 55	263·5	3·02	10·0	12·2	2	·018	
736	...	1 12·8	39 46	66·5	6·83	10·5	11·5	1	·756	
737*	...	47·8	30 0	140·5	3·37	11·0	12·0	2	·018	
738	...	2 9·9	29 29	259·2	6·32	10·0	13·0	2	·070	
739	...	10·2	30 16	132·5	6·70	9·5	11·0	2	·067	
740	...	10·2	29 12	336·7	5·13	10·0	12·2	2	·072	
741	+29·383	10·8	30 3	321·1	6·16	9·5	10·0	2	·067	AB
				142·3	19 ±	C = 9·9		2	·067	AC
742	...	57·0	29 55	5·4	9·38	10·0	11·0	2	·070	
743	...	59·0	28 33	228·6	6·70	9·5	10·0	2	·093	
744	...	3 0·5	30 4	253·5	7·62	9·5	10·0	2	·072	
745	...	7·5	29 16	357·5	4·37	10·0	11·2	3	·051	
746*	...	4 47·1	29 24	158·5	5·67	11·0	11·0	2	·118	
747	+29·752	47·3	29 41	265·5	2·14	9·5	9·6	2	·075	
748	...	49·1	29 26	27·2	4·75	11·0	11·4	2	·151	
749	...	5 44·7	29 37	105·8	3·98	9·5	11·5	1	·071	
750*	...	58·6	29 13	235·7	7·35	9·6	9·7	1	·183	
751*	...	6 31·8	28 31	227·4	5·36	10·0	12·0	1	·145	
752	+29·1499	7 11·9	29 37	355·4	5·36	9·5	11·5	1	·071	
753	...	17 45·9	37 47	349·9	7·10	10·2	11·5	4	·669	
754	...	18 16·3	39 12	52·6	4·16	10·0	11·5	2	·682	
755	...	33·3	38 54	113·5	6·81	11·0	11·0	2	·678	
756	...	52·8	28 8	327·4	3·27	10·2	11·7	3	·600	AB
				288·1	4·79	C = 13·2		2	·619	BC
757	...	55·3	32 27	135·8	4·06	10·2	10·6	2	·619	

Milburn No.	B.D.	1900 R.A.	1900 Decl.	P.A.	Dist.	Mags.	Nights	Date 1932+	
	°	h m	° ′	°	″				
758	...	18 57·4	+37 5	26·9	6·35	9·6 12·7	3	·771	AB
				146·6	7·76	C = 10·0	2	·758	AC
759	...	19 21·8	32 24	308·0	3·56	10·8 11·0	2	·619	
760	...	21·9	32 22	191·0	4·25	9·7 10·3	2	·619	
761	...	23·9	37 38	22·7	8·03	9·8 10·0	2	·779	
762	...	24·0	37 29	17·8	5·52	10·2 12·7	2	·779	
763	...	24·0	37 34	69·5	6·81	10·5 11·0	4	·817	
764	...	24·1	37 34	122·1	7·50	10·0 10·2	2	·779	
765*	+28·3419	34·2	28 59	232·7	5·67	9·3 9·8	2	·731	
766	...	46·5	38 17	352·0	5·79	10·5 10·6	2	·745	
767	...	49·5	38 40	120·4	7·29	10·2 10·5	2	·745	
768	...	20 4·9	39 18	109·1	4·35	10·0 10·1	2	·753	
769	...	5·2	38 57	130·3	4·25	11·5 13·0	3	·814	
770	+38·3920	5·3	38 47	288·6	4·49	9·5 12·5	3	·771	
771	...	5·5	38 49	19·9	2·87	9·5 11·5	4	·766	
772*	...	17·3	32 45	147·4	7·66	10·0 11·5	3	·653	
773	...	19·2	38 10	290·4	7·45	10·5 11·7	2	·684	
774	+27·3738	22·1	28 0	64·4	39 ±	8·0 10·5	1	·564	AB
				273·4	4·06	10·5 10·6	3	·600	BC
775*	+38·4116	25·9	38 36	164·2	9·68	8·8 10·5	2	·840	
776	...	25·9	38 41	77·1	9·56	9·5 9·7	2	·840	AB
	Sch 1137	...	...	309·8	19 ±	C = 9·5	2	·840	AC
777	...	31·4	38 53	186·5	5·29	11·0 11·1	2	·840	
778	...	34·5	39 17	309·8	4·68	10·0 11·5	3	·814	
779*	...	47·6	39 22	308·4	7·75	10·5 10·5	2	·731	
780	...	50·8	38 51	66·2	8·56	10·7 10·8	2	·684	
781	...	51·1	38 50	295·7	7·52	10·6 10·6	2	·709	
782	...	51·4	38 52	341·7	8·30	9·9 11·0	3	·671	AB
	Sch 1313	...	...	258·7	26 ±	C = 10·0	1	·673	AC
783	...	51·7	38 46	244·8	8·03	9·5 12·5	2	·747	
784	...	51·8	38 48	23·7	4·35	11·0 11·1	3	·767	
785	...	51·8	38 33	3·7	5·06	11·0 11·9	2	·840	
786	...	21 34·5	39 15	214·8	4·45	10·0 10·4	2	·802	
787	...	36·2	38 31	350·8	4·33	10·0 11·0	2	·871	
788	...	46·8	38 50	54·0	4·20	10·0 10·1	2	·731	
789	...	54·9	38 31	359·8	8·85	10·5 11·0	2	·669	
790*	+37·4490	22 3·0	38 13	328·6	9·51	9·5 10·5	2	·935	
791*	...	3·1	38 37	123·1	3·85	9·8 9·9	2	·935	
792	...	3·2	38 12	213·0	4·14	11·0 12·5	2	·935	
793	...	4·1	38 50	210·0	5·37	9·8 10·3	3	·787	
794*	...	13·4	38 21	346·2	5·37	9·5 10·0	3	·850	
795	...	13·5	38 27	78·6	5·88	10·0 10·5	2	·827	
796	...	25·0	38 45	316·9	4·41	10·0 10·5	2	·802	
797	...	32·6	38 38	216·1	4·12	10·0 13·0	1	·955	
798*	...	34·9	38 55	273·9	5·41	10·5 11·5	2	·747	
799	...	42·9	38 45	300·6	4·06	10·2 11·5	3	·764	
800*	...	43·4	38 55	29·7	3·04	10·0 10·0	2	·747	
801	...	52·3	39 13	356·3	4·56	10·5 10·6	2	·936	
802	...	23 13·7	38 40	83·2	8·01	10·0 11·0	2	·782	
803	...	20·2	38 45	322·7	7·27	9·4 11·5	2	·936	
804	...	20·5	38 34	313·0	8·55	10·0 10·5	3	·906	
805	...	25·0	38 37	262·7	8·49	9·8 10·2	2	·747	
806	...	51·9	39 12	64·4	4·70	9·8 11·0	2	·747	

*Notes on stars marked **

735. At 113°·5 ; 156″ ± from B.D. +25°·150.
737. At 69°·7 ; 87″ ± from B.D. +29°·318.
746. At 93°·5 ; 66″ ± from B.D. +29°·750.
750. A star of 9·6 mag. at 3°·6 ; 37″ ±; 1932·183.
751. Discovered on 1928 January 11, but not measured.
765. A star of 9·3 mag. at 52° ± ; 116″ ± .
772. A star of 9·5 mag. at 308°·1 ; 31″ ± ; 1932·654.
775. B.D. +38°·4111 ; 8·5 mag. ; at 298°·0 ; 65″ ± ; 1932·840.
779. At 326°·5 ; 90″ ± from B.D. +39°·4348.
790. A star of 9·6 mag. at 160°·5 ; 48″ ± ; 1932·917.
791. A star of 9·9 mag. at 140°·1 ; 35″ ± ; 1932·917.
794. A star of 10·0 mag. at 73°·6 ; 27″ ± ; 1932·895.
798. At 327° ± ; 58″ ± from B.D. +38°·482.
800. A star of 9·5 mag. at 173°·1 ; 40″ ± ; 1932·739.

Missing Stars

	h m s	° ′		
B.D. +25·157	0 55 2·2	+25 42·2 (mag. 9·5).	1932 Jan. 8.	No star near this place. M.
+35·667	3 10 47·4	+35 10·0 (mag. 9·5).	1932 Jan. 26.	Not seen. E.
+29·578	3 27 0·8	+29 55·7 (mag. 9·5).	1932 Jan. 8.	Missing. Queried in B.D. M.
			1932 Jan. 26.	No star near this place.

FAINT DOUBLE STARS WITH COMMON PROPER MOTION IN THE SOUTHERN SKY.

W. J. Luyten.

1. Some years ago,* I gave a list of double stars, found with the blink microscope, for which the physical connection was evident from their large common proper motion. At the present time, somewhat more than half of the south polar area (south of $\delta = -60°$) has been examined, and more than 100 pairs with common proper motion found.

Before presenting and analyzing the observational material, it may be of value to make some theoretical deductions as to what may be anticipated about these stars, especially concerning the probable period of revolution and the expected orbital motion.

2. In a recent paper,† I have given a formula for the period to be expected in a "fixed" system of known separation, magnitude and parallax. The derivation of that formula seems to be in error. Without repeating the entire argument presented there, it will suffice to say that in the calculation

* *H.B.*, 852, 1927.
† *Proc. Nat. Acad. Sci.*, **16**, 252–256, 1930.

of the mean value of the radius vector in a Keplerian ellipse as a function of a and e, the *time* should have been used as independent variable. This then gives the well-known relation :

$$\bar{r} = a\left(1 + \frac{e^2}{2}\right).$$

The quantity actually required, however, was not the mean radius vector, but the mean of its logarithm, which may be found from

$$\overline{\log r} = \frac{1}{P}\int_0^P \log r\,dt,$$

yielding, as is easily verified,

$$\overline{\log r} = \log a + \mathrm{mod}\left[\frac{e^2}{4} + \frac{e^4}{32} + \frac{e^6}{96} + \ldots + \frac{e^{2n}}{(2n)^2}\,\frac{1\cdot 3\cdot 5\ldots(2n-3)}{2\cdot 4\cdot 6\ldots(2n-2)}\right].$$

The value of $\overline{\log r}$ calculated in this way is slightly less than that used before ; if the part depending on e is denoted by $f(e)$, it is seen that in the formula for $\log P$ it enters as $-\frac{3}{2}f(e)$. Hence the new value of $\log P$ will be larger than that found before, and should read :

$$\log P = 1{\cdot}460 \log d - 0{\cdot}487 \log \text{mass} + 0{\cdot}22 \pm 0{\cdot}35, \qquad (1)$$

instead of :

$$\log P = 1{\cdot}460 \log d - 0{\cdot}487 \log \text{mass} + 0{\cdot}17 \pm 0{\cdot}35$$

(formula (14) of the paper referred to above).

When the formula is applied to the calculation of the periods of 15 binaries with known periods,* nearer than 10 parsecs, it is found that

the mean value of $\log P$ from (1) is 1·88,
,, ,, ,, observed is 1·89.

The agreement is entirely satisfactory and considerably better than with the old formula which gave 1·83.

Van den Bos has called my attention to the fact that the statistically derived relation between P and e, used in the derivation of the final formula above, may largely, if not entirely, be based upon selection, and it might be possible that better values for $\log P$ are obtained by simply assuming the eccentricity to have the average value, say, 0·667, as according to Jeans.

This would change formula (1) to :

$$\log P = \tfrac{3}{2} \log d - \tfrac{1}{2} \log \text{mass} + 0{\cdot}120$$

or, if the term dependent upon the eccentricity is retained as such,

$$\log P = \tfrac{3}{2} \log d - \tfrac{1}{2} \log \text{mass} + 0{\cdot}200 - \tfrac{3}{2}f(e). \qquad (2)$$

3. These rather simple formulæ are not directly applicable to the problem in hand, since for the present lists of stars, the parallaxes and hence the linear separations, absolute magnitudes and masses are not known. The only known quantities are : the apparent magnitudes, the angular

* *Loc. cit.*, **16**, 258, 1930, Table I.

separation and the annual (angular) proper motion; the problem then is to derive from these the period and orbital motion to be expected.

Let P = period
a = semi-axis major } of the relative orbit,
e = eccentricity
G = total mass,
r = linear radius vector of the true orbit,
v = ,, velocity in the orbit,
ψ = angle between line of sight and radius vector,
θ = ,, ,, ,, ,, velocity,
d = observed, foreshortened, linear distance,
u = ,, ,, ,, velocity.

Let it further be assumed that the angle θ is completely independent of ψ (which is not strictly true), and hence that the frequencies of both ψ and θ are proportional to their sines. Introducing Kepler's eccentric anomaly E, we then have

$$u = v \sin\theta, \qquad d = r \sin\psi, \qquad r = a(1 - e\cos E),$$

$$\frac{2\pi}{P}dt = (1 - e\cos E)dE.$$

Since

$$v^2 = \kappa^2 G\left(\frac{2}{r} - \frac{1}{a}\right),$$

where $\kappa = 2\pi$ when the solar mass, astronomical unit and year are used as units, we have:

$$u = 2\pi\sqrt{\frac{G}{d}} \cdot \sqrt{\sin\psi(1 + e\cos E)} \cdot \sin\theta, \tag{3}$$

and

$$\bar{u} = 2\sqrt{\frac{G}{d}}\int_0^{\pi/2} d\psi \int_0^{\pi/2} d\theta \int_0^{\pi} dE[(\sin\psi)^{3/2} \cdot \sin^2\theta \cdot \sqrt{1 + e\cos E(1 - e\cos E)}];$$

$$\bar{u} = \frac{\pi}{2}\sqrt{\frac{G}{d}} \cdot I_1 \cdot I_2,$$

where

$$I_1 = \int_0^{\pi/2} (\sin\psi)^{3/2} d\psi, \tag{4}$$

$$I_2 = \int_0^{\pi} \sqrt{1 + e\cos E} \cdot (1 - e\cos E)dE.$$

Both integrals are elliptical; numerical quadrature gives $I_1 = 0{\cdot}873$, while I_2 can be solved by expanding the integrand into a power series in e, subsequent integration then gives:

$$I_2 = \pi\left[1 - \frac{5e^2}{2^4} - \frac{39e^4}{2^{10}} - \frac{245e^6}{2^{14}} - \frac{29 \times 33 \times 35e^8}{2^{22}} - \frac{13 \times 21 \times 33 \times 37e^{10}}{2^{26}} \cdots\right]$$
$$= \pi \cdot \phi(e).$$

The values of $\phi(e)$, $\log \phi(e)$ which equals $-\frac{3}{2}f(e)$ from formula (2),* are given in Table I. For the range of e given, these may be replaced approximately by

$$\phi(e) = 1 - 0{\cdot}354e^2$$
$$\log \phi(e) = 0{\cdot}005 - 0{\cdot}188e^2.$$

Table I

e	$\phi(e)$	$\log \phi(e)$
0·4	0·94	−0·027
0·5	0·91	−0·041
0·6	0·87	−0·061
0·7	0·82	−0·086
0·8	0·77	−0·114
0·9	0·71	−0·149

We now have

$$\bar{u} = \frac{\pi^2}{2} \cdot 0{\cdot}873 \cdot \sqrt{\frac{G}{d}} \cdot \phi(e),$$

$$\bar{u} = 4{\cdot}30 \cdot \phi(e) \cdot \sqrt{\frac{G}{d}}. \qquad (5)$$

Since we neither have, nor want d and u, but s and w, the observed angular separation and angular orbital motion, we may write, introducing the parallax p,

$$w = up, \qquad s = dp,$$

w being expressed in seconds of arc per annum, and hence

$$w = 4{\cdot}30 \cdot \phi(e) \cdot G^{1/2} \cdot s^{-1/2} \cdot p^{3/2}, \qquad (6)$$

or, taking the logarithm,

$$\log w = 0{\cdot}633 + \log \phi(e) + \tfrac{1}{2} \log G - \tfrac{1}{2} \log s + \tfrac{3}{2} \log p\,; \qquad (7)$$

performing a like substitution in formula (2), we have :

$$\log P = 0{\cdot}200 - \tfrac{3}{2}f(e) - \tfrac{1}{2} \log G + \tfrac{3}{2} \log s - \tfrac{3}{2} \log p. \qquad (8)$$

Naturally, these two formulæ satisfy the relation

$$\log Pw = \log s + \text{const.},$$

since Pw is the average circumference of the apparent orbit, s the average radius vector. In the way the formulæ are derived here, the constant equals $0{\cdot}83 + 2 \log \phi(e)$.

4. Formulæ (7) and (8) are as yet not usable, since neither the masses nor the parallaxes are known. Although the legitimacy of the procedure

* The series developments for $\varphi(e)$ and $f(e)$ are totally different, owing to the fact that in the one the logarithm was taken *before* expansion and integration, in the other *afterwards*. The numerical values of $\log \varphi(e)$ and $\frac{3}{2}f(e)$ are, however, identical.

may well be questioned, we may attempt a plausible solution by making use of the following relations which have been derived statistically :

$$\log G = a + bM, \qquad \log p = c + dm + h \log \mu, \tag{9}$$

where $M = m + 5 + 5 \log p$, μ is the proper motion, and m and M are the combined apparent and absolute magnitude of the system. We then have, instead of (7) and (8),

$$\log w = 0{\cdot}633 + E - \tfrac{1}{2} \log s + A + Bm + C \log \mu, \tag{10}$$

$$\log P = 0{\cdot}200 + E + \tfrac{3}{2} \log s - (A + Bm + C \log \mu), \tag{11}$$

where

$$E = \log \phi(e) = -\tfrac{3}{2} f(e),$$
$$A = \tfrac{1}{2}a + \tfrac{3}{2}c + \tfrac{5}{2}b + \tfrac{5}{2}bc,$$
$$B = \tfrac{3}{2}d + \tfrac{1}{2}b + \tfrac{5}{2}bd,$$
$$C = \tfrac{3}{2}h + \tfrac{5}{2}bh.$$

For single stars, the observations give :

$$a = +0{\cdot}40, \qquad b = -{\cdot}085.* \tag{12}$$

In a double star with identical components, the total mass would be twice that of a single star, hence log G would be 0·30 larger, but the absolute magnitude would be only $0^{m}{\cdot}75$ brighter. When the components differ more than 2 magnitudes in brightness, the magnitude of the system is practically the same as that of the brighter star, but the combined mass is considerably greater than that of the brighter star. The value (12) for a therefore requires a statistical correction if (9) is to be applied to the absolute magnitude and mass of a binary system ; the amount of this correction may be roughly estimated as +0·22.

Hertzsprung,† however, finds empirically :

$$a = +{\cdot}42, \qquad b = -{\cdot}09,$$

and we therefore adopt

$$a = +{\cdot}52, \qquad b = -{\cdot}085.$$

With the formula for log p, the correction for the change from single stars to binary systems is small, and probably amounts to less than −0·02 in the value of c.‡

The adoption of the correct numerical values for c, d and h presents greater difficulties than in the case of a and b. Kapteyn originally found

$$c = 0{\cdot}690, \qquad d = -0{\cdot}0713, \qquad h = +0{\cdot}645. \tag{13}$$

I have shown that when the reduced proper motion, $H = m + 5 + 5 \log \mu$, is introduced, most mean-parallax formulæ may be replaced by the simpler

* *H.A.*, 85, 112, 1923 ; *cf.* also Russell, Dugan and Stewart, *Astronomy*, 2, 891.

† *B.A.N.*, 6, 57, 1930.

‡ This correction is opposite in sign and roughly of the same order of magnitude as that introduced by using constants determined for $\overline{\log p}$ instead of for $\log \bar{p}$; both will be neglected here.

one $M = n + qH$, containing only two constants. For the nearby stars, consisting mainly of dwarfs, discovered through their large proper motions, I found *

$$n = -2{\cdot}11, \qquad q = 0{\cdot}781.$$

For the very faint stars, such as those treated hereafter, no information is available concerning the relation between p and μ. Van Maanen and Willis † have recently published a valuable list of proper motions of 122 very faint stars. They derive the relation :

$$\log p = -1{\cdot}207 - 0{\cdot}011m + 0{\cdot}862 \log \mu \qquad (14)$$
$$\pm \cdot 133 \pm \cdot 008 \quad \pm \cdot 080$$

and comment on the smallness of the coefficient of m. But the parallaxes used in the determination of the constants were derived by first assuming that the mean peculiar velocity (radial or τ component) of these stars was equal to 50 km./sec. Thus they wrote $\bar{p} = 0{\cdot}0948\bar{\tau}$. But then it follows that also $\bar{\mu} = \frac{\pi}{2} \cdot \bar{\tau}$, and hence,

$$\bar{p} = 0{\cdot}0602\mu \quad \text{or} \quad \log \bar{p} = -1{\cdot}22 + \log \bar{\mu},$$

with the coefficient of m rigorously equal to zero and that for $\log \mu$ rigorously equal to unity. The deviations from this of the values derived by them may be explained by the fact that they had only nine equations of condition at their disposal, three of which represented rather discordant means of only a few stars.

Any deficiency from 1·00 for the value of h may be interpreted as an increase in linear tangential velocity with decrease in luminosity. Since at least part of this effect is due to selection, it is evident that for very faint stars the value of h should more and more approach unity. It is rather difficult, therefore, to say which value should be adopted here, though it is evident that it should be taken much larger than Kapteyn's value ; using the regression line rather than the line of symmetry derived from my own data, we may try :

$$c = -0{\cdot}60, \qquad d = -0{\cdot}050, \qquad h = 0{\cdot}76,$$

which yield :

$$A = -0{\cdot}725, \qquad B = -0{\cdot}107, \qquad C = +0{\cdot}98.$$

A set of slightly different values for c, d and h, equally well representative empirically, gives :

$$A = -0{\cdot}76, \qquad B = -0{\cdot}099, \qquad C = +1{\cdot}005.$$

Considering that the apparent magnitudes are not known with great precision for our stars here, we may well adopt, for the sake of simplicity :

$$A = -0{\cdot}74, \qquad B = -0{\cdot}100, \qquad C = 1{\cdot}00, \qquad (15)$$

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* *H.A.*, 85, 112, 1923.

† *Contrib. Mt. Wilson Obs.*, No. 412, 1930.

and thus write :

$$\log w = -0.11 + E - \tfrac{1}{2}\log s - 0.100m + 1.00 \log \mu, \qquad (16)$$

$$\log P = +0.94 + E + \tfrac{3}{2}\log s + 0.100m - 1.00 \log \mu. \qquad (17)$$

It should be emphasized that the simple fractional values for the coefficient of $\log s$ are a theoretical requirement, while those for m and $\log \mu$ should be regarded as statistical accidents. Indeed, since $C = \frac{3}{2}h(1 + \frac{5}{3}b)$, and since h may well have any value close to 1·00, all that we can say about C theoretically is that it must be less than $1.50(1 + \frac{5}{3}b)$ or less than 1·29.

5. Before using formulæ (16) and (17) for predicting purposes, it may be well to test them empirically. In the case of (17), we can simply use the same double stars as were used in testing (1), *i.e.* those binaries with well-known periods, nearer than 10 parsecs. Assuming the average value of 0·667 for e, we have $E = -0.08$, and (17) becomes

$$\log P = +0.86 + \log s + 0.100m - 1.00 \log \mu. \qquad (18)$$

The comparison is effected in Table II, where the different columns give, in order, the star's designation, the apparent visual magnitude of the system, the observed angular separation, the annual proper motion, the logarithm of the period (in years) as derived from the double-star orbit, as calculated from (1) and from (18).

Table II

Name	m	s''	μ	log P		
				Observed	From (1)	From (18)
			″			
85 Peg	5·9	0·72	1·29	1·43	1·48	1·12
ζ Her	2·9	1·2	0·60	1·54	1·56	1·49
α CMi	0·5	3·0	1·24	1·60	1·59	1·53
142 Sco	5·8	1·26	1·20	1·62	1·45	1·51
μ Her	8·9	0·74	0·82	1·63	1·48	1·64
Krü 60	9·4	1·36	0·87	1·65	1·11	2·08
α CMa	−1·6	10·6	1·32	1·70	2·12	2·12
ξ UMa	3·9	1·61	0·73	1·78	1·68	1·70
α Cen	0·0	7·02	3·68	1·90	1·49	1·57
70 Oph	4·1	6·56	1·13	1·94	2·37	2·44
41 Ara	5·5	3·60	0·97	2·11	2·37	2·26
ξ Boo	4·6	3·01	0·17	2·18	2·06	2·81
p Eri	5·3	9·22	0·29	2·34	2·63	3·38
o_2 Eri	9·4	4·52	4·08	2·39	2·32	2·17
η Cas	3·5	8·14	1·24	2·71	2·61	2·48
			Means	1·89	1·88	2·02

While formula (1) appears to give a satisfactory representation of the observations, the periods calculated from (18) seems to be systematically 30 per cent. too long. It may be remarked that equation (18) is based upon the relation between p and μ as found from stars selected principally by virtue of large proper motion. Table II contains 2 stars out of 15 which are exceptional in this connection, viz. ξ Bootis and *p* Eridani ; the former

might almost be called unique, since for it $p>\mu$, while for the latter μ/p is only 1·8 instead of 5 or 10 as usually. Excluding these two, we find for the mean value of log P for the remaining 13 stars :

1·83 (observed), 1·81 (from (1)), 1·86 (from (18)).

Since some of the longer periods in the observed list are somewhat uncertain—and uncertain orbital elements often give too small a period—the agreement may be considered as reasonably satisfactory.

6. If in formula (16) we assume the eccentricity to have the average value −0·667, we have :

$$\log w(\text{″ per annum}) = -0{\cdot}19 - \tfrac{1}{2}\log s - 0{\cdot}100m + 1{\cdot}00\log\mu. \qquad (19)$$

A comparison with observations is fortunately already available as Hertzsprung * has examined this relationship between w and μ for 27 double stars for which both these quantities are known. He has derived the empirical formula

$$\log w = -0{\cdot}20 - \tfrac{1}{2}\log s - 0{\cdot}1m + \log\mu \pm 0{\cdot}3. \qquad (20)$$

In view of what has been said about the uncertainties attached to the coefficients a, b, c, d and h, the agreement between (19) and (20) is satisfactory—fortuitously or suspiciously so. If in future it should become possible to determine a, b, c, d and h, or rather n and q with great certainty, then a comparison between (16) and (20) might give information concerning the average eccentricity in the orbits of these wide pairs.

7. Formulæ (18) and (19) are based upon visual magnitudes, while in what follows they will be applied to some exceedingly faint stars for which only the photographic magnitudes are known. Since these stars are probably all dwarfs of late spectral classes, we may assume the average colour-index to be +1·3, hence reduced to photographic magnitudes, formulæ (18) and (19) become, finally :

$$\log P = +{\cdot}73 + \tfrac{3}{2}\log s + 0{\cdot}100m - 1{\cdot}00\log\mu, \qquad (21)$$

$$\log w = -{\cdot}06 - \tfrac{1}{2}\log s - 0{\cdot}100m + 1{\cdot}00\log\mu. \qquad (22)$$

8. The observational material available consists of a number of stars found with the blink microscope to possess common or nearly common motion. In many such cases, the physical connection is so evident when looking through the blink as to leave no room for argument, even though distortion of the images or magnitude effects make the measured motions sensibly different. In other cases, where the measured motions differ no more than could be expected from the errors of measurement, the separation is so large as to make connection questionable. Since many uncertain factors enter into this, such as the direction of motion—with or against the solar motion—the final decision must be largely a matter of the observer's judgment, and could not now be expressed in numerical form.

Among the pairs found thus far, 73 appear to be physically connected with reasonable certainty ; the data for these stars are given in Table III.

* *B.A.N.*, **6**, 57, 1930.

For 29 further stars there still remained some doubt arising from various sources, as, *e.g.*, sometimes the faint companion was almost invisible and certainly not measurable on one of the plates—for these stars, the data are given in Table IV.

The arrangement of the two tables is identical and the columns give, successively: the serial number, the right ascension and declination for 1900, the apparent photographic magnitudes of the components, the rough direction, by quadrant only, of the fainter star relative to the primary, the angular separation, the annual proper motion and its position angle; and finally, the logarithm of the period (in years) and the expected orbital motion in seconds of arc per annum, both calculated from formulæ (21) and (22). For purposes of calculation, even the faintest magnitudes are often given to tenths and the proper motion always to $0''.001$, the uncertainties, however, being at least $0^m.5$ and $0''.02$.

There are several well-known stars in the list, such as κ Reticuli ($\wedge$ 9), $\wedge$ 53 which was announced by Innes many years ago, and $\wedge$ 71 for which meridian proper motions have been known for some time though the values for these proper motions were sufficiently different to prevent its recognition as a double. The reason that these stars have been given $\wedge$ numbers as well as those found by the writer is not because of any desire to claim credit for their "discovery"—anyone familiar with the blink microscope realizes that for most of the stars listed it would require skill *not* to discover them. Ultimately these stars will all be published in a general catalogue of proper-motion stars, but until that time it might be convenient to refer to them by means of a short designation. The symbol $\wedge$ does not yet appear to have been used by double-star observers, and is accordingly suggested for use here.

TABLE III

$\wedge$	R.A.	Dec.	m_1	m_2	q	s	μ	ψ	log P	w
	h m	° ′				″	″	°		″
1	0 09·8	−62 39	13·3	16·0	*sf*	6	0·184	94	3·95	0·0032
2	10·9	−68 33	13·3	15·0	*np*	48	·623	104	4·74	41
3*	38·8	−85 58	9·2	12·9	*n*	22	·099	247	4·67	22
4*	1 10·2	−71 29	12·5	13·5	*np*	22	·345	75	4·43	38
5	24·4	−61 31	12·0	13·0	*sf*	73	·169	53	5·46	12
6*	38·3	−68 10	8·5	13·5	*sp*	22	·575	147	3·83	151
7	55·6	−65 33	11·0	13·0	*p*	26	·243	34	4·55	34
8	2 23·7	−62 32	14·8	14·9	*sf*	144	·156	104	6·18	4^5
9*	3 27·6	−63 18	4·8	10·6	*sf*	54	·520	45	4·09	204
10	32·8	−68 02	11·0	12·0	*np*	13	·200	94	4·17	41
11*	4 00·8	−61 38	6·8	12·2	*sp*	29	·335	74	4·08	112
12	15·3	−73 41	13·4	14·7	*sf*	25	·219	56	4·80	19
13	16·0	−76 45	14·6	16·3	*np*	325	·199	37	6·64	3^5
14	19·3	−66 36	11·6	13·5	*sf*	32	·140	48	4·98	16
15	23·0	−67 36	13·8	13·8	*sf*	14	·107	9	4·73	12
16	45·8	−70 38	13·5	13·5	*sf*	93	·180	321	5·71	8^5
17	5 12·4	−67 15	12·3	12·7	*np*	18	·100	23	4·79	14
18*	22·5	−81 31	10·5	10·7	*s*	38	·172	331	4·85	25
19	6 00·2	−60 47	15·0	16·0	...	10	·350	134	4·17	32

Λ	R.A.	Dec.	m_1	m_2	q	s	μ	ψ	log P	w
	h m	° ′				″	″	°		″
20	6 07·4	−63 29	12·5	13·5	*s*	13	0·096	136	4·64	0·0014
21	37·2	−75 33	11·0	12·0	*nf*	25	·382	306	4·32	56
22	7 30·7	−63 40	13·1	13·6	*nf*	198	·208	12	6·12	7^{1}
23*	57·8	−63 22	10·4	12·4	*sp*	1050	·375	337	6·71	9^{6}
24	8 18·6	−67 29	12·8	14·7	*sp*	34	·258	148	4·87	21
25	36·5	−66 59	15·2	15·4	*sf*	16	·179	150	4·73	14
26*	8 53·0	−63 19	10·0	13·0	*nf*	125	·183	226	5·61	15
27*	55·9	−79 56	11·2	12·8	*s*	37	·355	307	4·63	41
28*	59·4	−59 57	11·8	11·8	*s*	5	·139	343	3·75	42
29*	9 09·1	−60 41	11·6	12·8	*np*	145	·183	321	5·84	10
30	18·0	−64 04	12·0	13·0	*f*	9	·245	78	3·94	48
31*	45·9	−80 52	9·9	9·9	*nf*	36	·089	164	5·03	16
32*	11 11·8	−62 42	11·6	12·3	*sp*	49	·045	305	5·73	4^{3}
33	12·9	−62 46	12·1	13·3	*n*	15	·077	219	4·79	12
34	39·6	−64 42	12·8	13·6	*n*	11	·322	230	4·02	49
35	12 43·0	−83 53	12·4	14·5	*sp*	230	·175	276	6·26	5^{9}
36	44·4	−61 28	15·0	15·5	*nf*	41	·091	60	5·64	44
37*	15 46·9	−83 57	9·2	13·3	*f*	22	·351	278	4·12	7^{8}
38*	51·9	−83 51	9·7	14·6	*nf*	17	·192	90	4·26	44
39	16 50·3	−70 22	11·8	12·8	*sf*	137	·094	50	6·11	5
40	52·4	−72 10	11·6	12·8	*s*	18	·068	187	4·91	10
41*	17 11·5	−66 16	10·2	14·0	*sp*	172	·101	201	6·10	6^{5}
42	36·1	−77 46	15·0	16·0	*nf*	30	·335	182	4·89	20
43	18 09·0	−66 39	11·5	11·8	*sp*	89	·135	191	5·61	10
44	09·3	−68 00	15·5	17·0	*sp*	18	·113	179	5·09	7
45	10·8	−61 57	11·5	17·0	*sp*	63	·396	208	4·98	31
46	28·5	−72 06	17·0	17·5	*nf*	380	·156	196	7·06	16
47*	41·3	−74 30	14·0	16·0	*sf*	6	·335	158	3·75	50
48	19 07·5	−70 33	15·0	16·5	*np*	94	·106	184	6·15	3
49	38·0	−67 00	17·2	17·6	*sp*	40	·036	177	6·25	1
50*	38·6	−68 25	18·0	18·5	*p*	3	·344	148	3·66	31
51	19 44·9	−67 36	14·2	15·2	*np*	18	·069	143	5·16	6
52	45·0	−67 41	16·0	17·3	*sf*	12	·134	353	4·82	8^{5}
53*	54·6	−65 52	13·0	14·3	*p*	16	·865	174	3·88	98
54	20 00·0	−86 12	11·6	14·2	*sf*	16	·071	213	4·84	11
55	17·0	−71 20	14·4	17·6	*sp*	144	·102	148	6·39	2^{8}
56	36·2	−63 41	16·0	17·0	*sf*	182	·133	175	6·57	2^{3}
57	45·0	−64 10	15·5	17·0	*sp*	8	·122	54	4·53	11
58	21 00·7	−68 18	15·5	16·0	*s*	4	·073	164	4·27	10
59	02·2	−67 48	15·0	17·2	*nf*	68	·097	146	5·98	3^{3}
60	16·0	−62 39	15·3	15·6	*nf*	15	·137	214	4·83	10
61	24·6	−65 47	16·5	16·5	*nf*	200	·207	161	6·45	3^{3}
62	45·2	−69 50	17·2	17·5	*nf*	51	·088	166	6·01	2^{3}
63	48·3	−71 36	14·0	14·1	*sp*	10	·233	131	4·19	30
64	22 16·7	−65 42	11·7	14·0	*np*	69	·165	115	5·43	12
65	25·9	−67 38	15·0	16·2	*sp*	120	·074	105	6·47	1^{9}
66	36·0	−65 52	16·0	17·0	*np*	20	·133	207	5·13	6^{9}
67	46·8	−69 05	14·2	14·6	*nf*	16	·081	302	5·00	7^{4}
68	23 06·5	−66 36	16·0	16·0	*s*	20	·069	116	5·22	5^{6}
69	06·6	−67 17	13·5	17·5	*nf*	15	·255	172	4·44	26
70*	08·9	−64 07	9·8	14·0	*sp*	24	·119	103	4·70	22
71*	11·0	−67 28	9·5	9·6	*nf*	71	·544	138	4·67	71
72	11·0	−60 45	13·5	16·0	*np*	92	·210	122	5·69	8^{7}
73*	43·4	−68 30	14·1	14·5	*f*	35	·170	92	5·19	10

TABLE IV

Λ	R.A.	Dec.	m_1	m_2	q	s	μ	ψ	log P	w
	h m	° ′				″	″	°		″
74	3 43·8	−78 04	12·6	15·5	*sp*	245	0·112	31	6·51	0·00036
75	5 10·0	−71 36	15·0	15·0	*sf*	177	·076	184	6·67	18
76	6 16·0	−62 04	14·8	14·8	*sf*	313	·110	61	6·84	21
77	7 51·0	−81 50	11·2	13·1	*sp*	90	·067	244	5·95	47
78	9 05·4	−60 12	14·5	15·3	*sf*	8	·061	283	4·61	91
79*	07·3	−60 06	9·0	12·0	*np*	112	·078	345	5·80	83
80	16·4	−60 34	14·0	14·3	*np*	187	·058	347	6·72	17
81	22·4	−63 13	13·0	14·5	*sf*	114	·035	304	6·55	15
82	32·9	−62 45	12·2	12·8	*sf*	265	·100	296	6·54	36
83	33·6	−62 51	14·0	14·0	*sf*	97	·066	318	6·21	28
84	44·4	−62 34	16·0	17·0	*sf*	39	·034	173	6·16	13
85	10 58·0	−63 16	12·8	13·0	*sp*	205	·095	272	6·43	36
86	11 02·0	−86 01	13·7	13·8	*f*	193	·098	306	6·47	31
87*	17 26·5	−82 09	9·6	13·5	*np*	170	·172	194	5·80	126
88	56·0	−84 44	15·0	15·7	*sp*	95	·097	209	6·16	31
89	18 09·6	−73 32	16·0	17·0	*nf*	153	·162	211	6·37	30
90	26·6	−69 01	13·0	15·0	*nf*	1065	·104	184	7·53	15
91	46·8	−78 48	14·5	14·5	*sf*	76	·158	156	5·73	66
92	53·0	−83 48	10·5	12·5	*np*	9	·129	323	4·08	347
93*	19 18·1	−70 58	14·5	17·0	*s*	121	·156	189	6·10	45
94	43·5	−66 21	17·1	17·4	*sp*	420	·069	195	7·48	06
95	46·4	−67 30	15·6	17·0	*np*	246	·117	171	6·79	19
96	52·0	−84 36	12·2	13·4	*sf*	140	·085	125	6·21	41
97	20 07·9	−71 30	15·5	17·0	*sp*	306	·116	159	6·92	17
98	46·4	−66 13	12·8	15·5	*nf*	240	·072	161	6·71	22
99*	21 08·4	−80 32	8·4	11·6	*nf*	164	·216	120	5·56	214
100	52·3	−68 29	13·5	15·0	*sf*	186	·183	138	6·20	55
101	23 09·1	−67 14	15·5	15·5	*nf*	121	·074	85	6·46	20
102	19·1	−67 28	14·5	15·0	*sf*	30	·051	140	5·64	32

*Remarks on stars marked **

3. C.P.D. −86° 8.
4. The value for the proper motion given in H.B. 852 is wrong; see also H.C. 327.
6. C.P.D. −68° 77.
9. κ Reticuli.
11. C.P.D. −61° 295.
18. C.P.D. −81° 135.
23. C.P.D. −63° 861.
26. C.P.D. −63° 1062.
27. C.P.D. −79° 363.
28. A 8·5 star, *np* 4″·5 (C.P.D. −59° 1227), *may* have the same motion, but magnitude effect makes it impossible to measure this.
29. C.P.D. −60° 1342.
31. C.P.D. −80° 386 and 388.
32. C.P.D. −62° 1945.
37. C.P.D. −83° 593.
38. C.P.D. −83° 596.
41. C.P.D. −66° 3089.
47. This pair may possibly show orbital motion, see remarks at end of paper.
50. Some orbital motion is suggested here, see remarks at end of paper.
53. Found by Innes.
70. C.P.D. −64° 4331.
71. C.P.D. −67° 3959 and 3960.
73. A 9^m star, *fo* 3″·3 (C.P.D. −68° 3584), *may* have the same motion, but magnitude effect makes it impossible to measure this accurately.
79. C.P.D. −59° 1296.
87. C.P.D. −82° 711.
93. The companion is so faint as to leave some doubt about its existence.
99. C.P.D. −80° 1006.

The magnitudes have been reduced to the international photographic scale as well as possible, but the writer does not feel convinced that systematic errors have been eliminated; in particular the magnitudes of the very faint stars might well have been recorded too faint. Systematic errors in the proper motions are probably not statistically serious, but the parallaxes implied through the use of constants *c*, *d* and *h* may well be too small, as many of these exceedingly faint proper-motion stars may well be intrinsically faint rather than of high speed. The combined effect of these causes would be to give the periods too long and the orbital motions too small, but, in any case, this seems the safer side to err on.

Where the influence of observational selection is so glaringly uncertain, any interpretation of the results cannot have too much significance beyond the obvious conclusion that double stars with small separations have little chance of being detected on the plates used, while pairs with large separations or small proper motions escape being recognized as doubles.

The 102 stars listed here represent roughly about half of the area south of declination $-60°$, and hence in the entire sky some 3000 pairs of this type may be expected as against some 25,000 ordinary visual pairs.

The visual double-star observer may not consider these stars as genuine doubles. They certainly fall far outside the limits in magnitude and separation ordinarily set. On the other hand, common proper motion, provided this can be determined accurately enough, is a better criterion of physical connection than mere propinquity in the sky. Also, it should be remembered that a large proportion of visual doubles are giants and stars of early spectral types, while the double stars with common proper motion are overwhelmingly dwarfs, and as such much more representative of the stars in space.

It must be admitted that there is little likelihood of determining actually the orbital motion in any of these stars—with only one or two exceptions, but this very fact should make it easier to decide whether the stars form physical doubles. The scale of the plates used (1 mm. = 60″) as well as their large area with consequent distortion over a large part of the plate does not permit accurate measurement. For this reason, the relative position has been given only very roughly as, *e.g.*, *np* : 10″. It is hoped, however, that the more powerful instruments now in operation or under construction in the southern hemisphere will eventually be able to attack this problem.

9. In conclusion, two stars may be mentioned specially which appear to merit individual attention, since in the blink microscope a suggestion of orbital motion was seen.

$$\wedge\ 47 \qquad 18^{h}\ 41^{m}\cdot 3, \qquad -74^{\circ}\ 30'.$$

Measures on the two available plates gave

4″·5	110°	in 1900,
6″·5	145°	in 1929,

but the pronounced distortion of the images and the strong magnitude effect make these measures highly uncertain. If they are roughly correct,

then the period of the system should be of the order of 300 years, and since the mass cannot very well be more than half that of the Sun, the parallax should be at least 0″·1.

$\wedge$ 50 $\quad 19^{h}\ 38^{m}.6 \quad -68°\ 25'.$

For this star, the images are exceedingly faint and hard to see—magnitude 18 and 18½—and one cannot even be certain that it *is* a double star. The blink microscope, however, very distinctly conveys the suggestion, not of a single image, but of a slight trail which not only "jumps" but also turns in orientation. Extremely uncertain measures gave :

5″·2	250°	1900·7,
2″·6	290°	1929·7.

If these are roughly correct, then the period would be about 250 years, the mean distance about 25 astronomical units (since the mass cannot very well be more than 0·3) and the parallax again about 0″·1. If this is so, then $\wedge$ 50 not only seems to be a faint replica of Krüger 60, but to possess as its components stars of an absolute magnitude fainter than 18—fainter than any now known.

Minneapolis, Minn. :
1932 *December* 5.